

GEORGIA INSTITUTE OF TECHNOLOGY  
OFFICE OF CONTRACT ADMINISTRATION  
SPONSORED PROJECT INITIATION

Date: 7/23/79

Project Title: Failure Processes in Advanced Composite Structures

Project No: E-16-672/E-20-667 (Continuation of E-23-637 which began 7/1/78)

Project Director: Dr. L. W. Rehfield/Dr. S. N. Atluri

Sponsor: Air Force Office of Scientific Research

Agreement Period: From 7/1/79 Until 6/30/80 (R & D Period)

Type Agreement: Contract No. F49620-78-C-0085

\$529,080 (Partially funded at \$327,163 through 9/30/79)

Amount:	AE	CE	ESM	Totals
AFOSR	\$231,900 (E-16-672)	\$38,145.68 (E-20-667)	\$57,117.32 (E-23-637)	\$327,163
GIT	10,240 (E-16-329)	134.29 (E-20-323)	15,265.71 (E-23-333)	25,640
TOTALS	<u>\$242,140</u>	<u>\$38,279.97</u>	<u>\$72,383.03</u>	<u>\$352,803</u>

Reports Required:

Annual Technical Report; Quarterly Progress Reports; Final Technical Report

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Defense Priority Rating: D0-C9 under DMS Reg. 1

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GEORGIA INSTITUTE OF TECHNOLOGY  
OFFICE OF CONTRACT ADMINISTRATION  
SPONSORED PROJECT TERMINATION

Date: 2/11/81

Project Title: Failure Processes in Advanced Composite Structures

Project No: E-16-672/E-20-667

Project Director: Dr. L.W. Rehfield/Dr. S.N. Atluri

Sponsor: AFOSR; Bolling AFB, D.C.

Effective Termination Date: 11/14/80

Clearance of Accounting Charges: 11/14/80 (Perf.)

1/14/81 (Rpts.)

Grant/Contract Closeout Actions Remaining:

- ☒ Final Invoice and Closing Documents
- ☐ Final Fiscal Report
- ☒ Final Report of Inventions
- ☐ Govt. Property Inventory & Related Certificate
- ☐ Classified Material Certificate
- ☐ Other \_\_\_\_\_

Assigned to: Aerospace Engineering/Civil Engineering (School/Laboratory)

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FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report  
1 July 1978 - 30 September 1978  
AFOSR Contract F49620-78-C-0085

L. W. Rehfield and S. N. Atluri  
College of Engineering  
Georgia Institute of Technology  
Atlanta, Georgia 30332

## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Engineering Science and Mechanics. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is a cooperative effort between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-East in St. Louis.

For convenience, this report contains a separate section devoted to each phase. Thus, activity associated with each phase is easily identified.

## PHASE A

"Computational Methods for Fracture and Fatigue Analyses"

S. N. Atluri

In the first three months (7/1/78-10/1/78) of the contract period, the following research was performed:

(i) Stress and Fracture Analysis of Angle-Ply Laminates: A general formulation for an efficient assumed stress type finite element method for the analysis of symmetric angle-ply laminates of the form  $(\pm\beta_1h_1/\pm\beta_2h_2/\pm\cdots/\pm\beta_nh_n)_s$  where  $\beta_n$  and  $h_n$  are, respectively, the orientation and thickness of the  $n^{\text{th}}$  angle-ply component of the laminate was developed. This newly developed method was kept general enough to account for the presence of through-thickness cracks and holes.

In this new assumed-stress finite element formulation, a fully three-dimensional stress state, including transverse shear and normal stresses are accounted for, and the interlayer traction reciprocity condition is satisfied a priori. The finite element, which is of a quadrilateral plan form, consists of the entire stack of lamina; in the presently developed form, each element can consist of 8 layers, while the formulation allows for easy modification to account for more layers if necessary. The procedure was demonstrated to be highly cost effective and efficient, in comparison with other methods reported in recent literature, in test cases where exact three-dimensional solutions for laminates are available.

Some of these results are being reported in a paper, "An Efficient Assumed-Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates," by T. Nishioka and S. N. Atluri, to be presented and published at the 15th Society of Engineering Science Meeting, University of Florida, Gainesville, Dec. 4-5, 1978.

Efforts have also been initiated into the development of special elements with embedded mixed-mode stress and strain singularities, whose intensities vary within each anisotropic layer. These elements are planned to be used in the fracture analysis of angle-ply laminates with through-thickness cracks. Preliminary results indicate the efficacy of the method in contrast to those presented in the very limited literature on the subject.

(ii) Two Dimensional Analysis of Heterogeneous Systems: Preliminary development has been carried out for a finite element methodology for two dimensional analysis of lamina where in the matrix and fibers are modelled separately. Account has been taken of cracks in the matrix at an arbitrary angle to the fibers.

(iii) Continuation of on-going Research into Fracture and Fatigue of Metallic Materials: Further analyses of crack-closure and delay effects on the retardation/acceleration of fatigue crack growth under Mode I type cyclic loading consisting of low-to-high and single overload were made. Results are currently being prepared for publication. Efforts are under way to analyse crack-closure effects, if any, under Mode II cyclic loading.

## PHASE R1

## "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

In the first three month period, the work described below was performed.

Hygrothermal Effects

The primary emphasis has been placed upon completing beam vibration tests. These experiments have produced nearly 2000 distinct data values over a two year period. Of primary interest is saturated, hot stiffness and damping data obtained at intermediate, elevated temperatures (180F, 140F) on graphite/epoxy composite laminated beams. A significant finding has emerged; the preliminary 200F experiments reported last year apparently correspond to conditions at or above the glass transition temperature of the resin.

An additional experimental study has been performed which has turned out to be of great significance. By virtue of some difficulties, experiments were conducted on aluminum beams. A closer examination of the resulting data prompted a comparative study of hygrothermal effects on graphite/epoxy with the effect of temperature alone on two aluminum alloys. The aluminum alloy beam tests have just been completed. The data indicate that damping changes associated with hygrothermal effects on the composites are of the same order of magnitude as those produced by elevated temperature for the aluminum alloys tested. The only new consideration for graphite/epoxy composites that must concern the designer is the increased stiffness reduction in matrix controlled modes of response.

The above work will be presented in two papers this fall. One is to be presented at the ASCE Annual Conference in Chicago in October, and the other at the ASME Winter Annual Meeting in San Francisco in December.

In addition to the experimental work, analytical studies of the influence of hygrothermal effects on the design of stiffness critical composite struc-



tures are being conducted. Specifically, beam vibration and panel buckling due to inplane loads are being considered. It is premature to discuss results at this time. The tentative impression obtained thus far is that the weight penalty due to hygrothermal effects is not too great, at least in the cases for which results have been obtained to data.

#### Dynamic Behavior of Woven Composites

Discussions on specimen designs have been conducted with the McDonnell Douglas Corporation in St. Louis. MDC personnel have indicated a willingness to supply the necessary specimens. This is the extent to which this task has been pursued in this three month period.

## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

In the first three month period, considerable progress in performing experiments has been made. The nature of the work is briefly summarized below.

Isogrid Element Tests

Buckling experiments on small and large flat panels were completed under the previous contract F49620-77-0077. The data taken has been studied further and a tentative conclusion reached. It appears that the critical dimension of an isogrid structure is rib width and that acceptance of a finished part may be specified in terms of it. The panel buckling results will be presented in an upcoming paper at the Fourth USAF/Army/Navy/NASA Conference on Fibrous Composite Structures. Also, another paper is being considered for presentation at the 20th AIAA Structures, Structural Dynamics and Materials Conference.

Two of the small flat panels which were previously tested nondestructively in compressive buckling were sectioned into two beam specimens each. The beams were cut such that a "backbone" rib is situated along the length, roughly at midwidth. Extensive testing of these beam elements is planned, much of which has been completed during this three month period. A summary of the experiments is as follows:

1. Buckling tests (nondestructive)
2. Stiffness evaluation (three-point bending tests (TPB tests) )
3. Stiffness evaluation (four-point bending tests (FPB tests) )
4. Stiffness evaluation (cantilever bending tests (CB tests) )
5. Flexural strength tests (TPB and FPB tests)

6. Beam vibration tests
7. Geometric characterization
8. Fiber content determination

Tests 1, 2, 3, 5, and 7 have been completed on the four beam specimens. Tests 4 and 6 have been explored. The necessary chemicals and equipment required to perform test 8 are currently being assembled.

The reason for the extensive, varied number of flexural stiffness tests on the beams is to assess the influence of transverse shear contribution to the flexibility. Due to the unidirectional nature of the woven ribs, transverse shear stiffness is rather low. Consequently, an experimental indication of the importance of this effect is sought.

#### Correlation of Experimental Results and Theory

A thorough assessment of test data in the light of theoretical design analysis predictions will eventually be made. In preparation for this, the McDonnell Douglas computer program H485 has been put "up" on the GIT Cyber computer. Furthermore, the basic equations have been verified by an independent development. This activity is a prerequisite for the correlation study.

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FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report  
1 October 1978 - 31 December 1978  
AFOSR Contract F49620-78-C-0085

L. W. Rehfield and S. N. Atluri  
College of Engineering  
Georgia Institute of Technology  
Atlanta, Georgia 30332

## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Engineering Science and Mechanics. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase. Thus, activity associated with each phase is easily identified.



## Phase A

### "Computational Methods For Fracture and Fatigue Analyses"

S. N. Atluri

In the previous three months (10/1/78 to 1/1/79) the following research was performed.

(i) Stress and Fracture Analysis of Angle-Ply Laminates:

Further results on stress analysis of uncracked multi-layer angle-ply laminates subjected to inplane as well as out-of-plane loads were obtained. The obtained results, especially for interlaminar stresses, provided the needed confidence in the efficiency of the developed analysis procedure, as reported in the previous progress report (10/1/78).

The above results for stress analysis were presented at (i) 4th USAF/Army/Navy/NASA Composites Review Conference in Dayton, Ohio, Oct. 31, 78, and (ii) 15th Annual meeting of the Society of Engineering Science, Univ. of Florida, Dec. 4-6, 1978, and were published in the paper, T. Nishioka and S. N. Atluri, "An Efficient 'Assumed-Stress' Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates", in Recent Advances in Engineering Science, (Ed. by R. L. Sierakowski) Univ. of Florida Press, pp. 95-101.

Development of "multi-layer hybrid crack element" for modelling the near-crack-tip region of cracks in angle-ply laminates has been continued. A new procedure to account for stress-free conditions, on the surface which intersects the crack front (in the case of through-the-thickness cracks in angle-ply laminates), and their effect on the variation of stress-intensity factors along the crack front, has been developed. The above procedures have been checked as to their accuracy in several test cases. Results have been obtained for the case of a through-the-thickness cracks in  $(0^\circ/90^\circ/90^\circ/0^\circ)$  and  $(-45^\circ/+45^\circ/+45^\circ/-45^\circ)$  laminates.

The above results for cracks in laminates are being presented in a paper, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed-Stress Finite Element Approach-I", AIAA Paper No. 79-0801, 20th AIAA/ASME/ASCE/AHS Structures, Dynamics and Materials Conference, St. Louis, Missouri, April 1979.

(ii) Two Dimensional Analysis of Heterogeneous Systems:

Computer programming aspects have been completed for two dimensional analysis of lamina wherein the matrix and fibers are modeled separately. Several test cases are currently being formulated.

(iii) Continuation of Ongoing Research into Fracture and Fatigue of Metallic Materials:

Analyses of Crack-closure phenomena in 4 cases of spectrum loading, constant amplitude, high-to-low, low-to-high, and single overload in an otherwise constant amplitude block loading were made. Definite conclusions regarding factors that cause growth acceleration, retardation, and delay were reached. An analysis of a center-cracked specimen under pure Mode II fatigue loading was made.

The rather extensive results developed in the above analysis of fatigue cracks are being presented in two papers: (i) M. Nakagaki and S. N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II", AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS Structures, Dynamics and Materials Conference, St. Louis, Missouri, April 1979 and (ii) "M. Nakagaki and S. N. Atluri, "Fatigue Crack Closure and Delay Effects Under Modes I and II Spectrum Loading", at Third International Conference on Mechanical Behaviour of Materials, University of Cambridge, Cambridge, U. K., August 1979.

Efforts have also been initiated into the analysis of fatigue behaviour of cracks near cold-worked fastener holes in thin sheets.

## PHASE R1

"Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

In the second three month period, the work described below was performed. Emphasis in this quarter has been upon Phase R2 rather than this phase.

Hygrothermal Effects

The second group of hygrothermal beam vibration tests has been completed. This group involved tests at 180F, 140F and 77F (Room Temperature) in the fully saturated state. Also, dry tests at 180F and 77F have been conducted to provide a contrast. All data obtained to date is being assembled and fully presented in Mr. R. P. Briley's Ph.D. thesis, which is in preparation. A modified version of this compilation will be prepared for publication as an AFOSR technical report. The extensive data base cannot be published in any other convenient way.

Portions of the above work were presented at two meetings during this three month period. The appropriate references are

Rehfield, L. W., and Briley, R. P., "Dynamic Environmental Effects on Aerospace Materials", ASCE Preprint 3328, presented at the ASCE Annual Convention, Chicago, Ill., October 16-20, 1978.

Rehfield, L. W., and Briley, R. P., "A Comparison of Environmental Effects on Dynamic Behavior of Graphite/Epoxy Composites With Aluminum Alloys", ASME Paper 78-WA/Aero-10, presented at the ASME Winter Annual Meeting, San Francisco, December 10-15, 1978.

It is appropriate at this point, when our group of experiments has been completed and when the AMFL program at General Dynamics-Fort Worth and the AFFDL program at Grumman are well underway, to evaluate the entire

picture pertaining to hygrothermal testing. One of our objectives in preparing the AFOSR report with all our data will be to study and report on what additional testing really needs to be done.

The analytical studies of the influence of hygrothermal effects on the design of stiffness critical composite structures are progressing rather slowly. The graduate student working on them, P. L. N. Murthy, has discovered background theoretical problems that must be addressed before the hygrothermal issue is considered. Emphasis is on buckling and postbuckling behavior of columns and plates and on beam vibrations.

#### Dynamic Behavior of Woven Composites

Panels from which beam specimens for vibration testing will be cut have been manufactured by McDonnell Douglas in St. Louis. They have been delivered to Georgia Tech, but, because of a lack of an experienced person to work on this task at present, the desired beam specimens have not been cut from the panels. A graduate student, who is an Air Force Officer, is being groomed for this work.

## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

In the second three month period, greater emphasis has been placed upon this phase of activity. This is because a clear indication was needed of the future requirements for this work. The work completed this quarter is summarized below.

Isogrid Buckling Tests

Considerable progress toward understanding the buckling behavior of the small and large flat panels that were tested under the previous contract has been made. This insight has been reported in the final AFOSR report submitted for contract F49620-77-C-0077. When looked at in the proper way, correlation of the experimental buckling data with existing theory is, on a relative basis, excellent. Inconsistency in manufacturing has been clearly identified as the primary reason for the scatter of the test data.

A major step has been taken, therefore, toward resolving the issues surrounding the potential use of continuous filament advanced composite isogrid in stiffness critical applications. This is not a small accomplishment in view of the scatter of the test data.

The above work was the basis for the following paper:

Rehfield, L. W. Deo, R. B., and Renieri, G. D., "Continuous Filament Advanced Composite Isogrid: A Promising Structural Concept", presented at the Fourth Conference on Fibrous Composites in Structural Design", San Diego, November 14-17, 1978.

Shear Deformation Theory

Another fact uncovered during the correlation study of the buckling



data is the fact that transverse shear deformation cannot be ignored. A simple, crude method of estimating its' effects was used in the correlation study. Since it is not a major undertaking, a shear deformation theory is currently being developed for the isogrid type of construction. The primary task here is developing a rational means for determining the transverse shear stiffness. The approach selected employs the determination of complementary strain energy.

### Isogrid Element Tests

While the panel buckling tests provided overall information on stiffness, good quantitative correlation with theory required additional stiffness tests, preferably on elements. Also, strength information is needed. Beam specimens were cut from the previously tested isogrid panels and tested in a number of ways, including buckling tests, three and four point bending tests and flexural strength tests. Many of these tests were completed in the first quarter on four beam specimens, with the remainder completed during this quarter.

What is needed is good data on rib and skin properties determined from separate tests. Rib and skin elements can be salvaged from the four beam elements already tested. This has been done, but difficulties in using resistance strain gages on braided and woven structures has hampered our testing. These difficulties must be resolved and reliable data taken. Rib and skin data will permit an assessment of the quantitative accuracy of isogrid theory. What is needed is a breakthrough of the same proportion as the one which permitted the buckling data to be understood.

All of the chemicals and glassware were assembled to perform fiber content determinations on resin matrix composites. Preliminary tests have been run on samples previously tested by McDonnell Douglas; our results agree very well with theirs. We are ready, therefore, to per-

form fiber content tests on isogrid rib and skin elements. These tests are also critical in evaluating the accuracy of the theory.

E-16-672  
E-23-637

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report # 2  
1 January 1979 - 31 March  
AFOSR Contract F49620-78-C-0085

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## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Engineering Science and Mechanics. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

## Phase A

### Computational Methods for Fracture and Fatigue Analyses

S. N. Atluri

In the previous three months (1/1/79 to 4/1/79) the following research was performed.

#### (i) Stress and Fracture Analysis of Angle-Ply Laminates:

Development of a "multi-layer hybrid crack-element" for modelling cracks in angle-ply laminates has been nearly completed. In this development, a new theoretical method to study the effects of free surfaces on stress-intensity factors has been formulated. In the analytical solutions for embedded flaws, in either isotropic or anisotropic media, it is found that the asymptotic (singular) stress solution obeys the plane-strain constraint. However, when a crack intersects a stress-free surface, say at a right-angle, it is noted that the solution in the vicinity of the free surface must necessarily be of a plane-stress type. The transition from this plane-stress state to a plane-strain state in the interior region of the crack, has been a subject of much controversy in literature. In the present research, the constraints of either plane-stress or plane-strain along the crack-border have been removed a priori. Instead, an asymptotic (singular) stress solution, for cracks in general multi-layer anisotropic media, that satisfies the equilibrium equations only a priori has been derived. The principle of complementary energy is then used to force the above stress-solution to satisfy compatibility of deformation. The numerical solution thus automatically predicts conditions of plane-stress or plane-strain along the crack border, as the case may be.



Excellent results for cracks in multi-layered anisotropic, or isotropic, laminates have been obtained.

Modifications to the above procedures, to enable the treatment of cracks near holes in adhesively-bonded metallic laminates, are currently being studied.

The following is a cumulative list of publications (since 7/1/78) and/or presentations under this item of research under the present contract.

- (i.1) "Cracks in Laminates," presentation, 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, Oct. 31, 1978.
- (i.2) T. Nishioka and S. N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Lamintes," in Recent Advances in Engineering Science (Ed. R. H. Sierakowski) Univ. of Florida Press, pp. 95-101.
- (i.3) T. Nishioka and S. N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite Element Approach, I," AIAA Paper No. 79-0801, Proc. 20th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials, Conference, St. Louis, Missouri, April 1979, pp. 315-326.
- (i.4) S. N. Atluri, M. Nakagaki, and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws," International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
- (i.5) T. Nishioka and S. N. Atluri, "Analysis of a Through Crack in ( $-45^{\circ}/+45^{\circ}/+45^{\circ}/-45^{\circ}$ ) Laminate Using a Complementary Energy Principle," paper being submitted to Journal of Engineering Fracture Mechanics, May 1979.

(ii) Two Dimensional Analysis of Heterogeneous Systems:

Several two dimensional problems of cracks in lamina wherein the matrix and fibers are modelled individually have been solved. The general case of the crack running at an angle to the fiber orientation was treated. Results for stress-intensity factors that are in conformity with the anticipated physical behaviour have been obtained. It is noted that similar results do not appear in prior literature. These results are currently being prepared for publication.

(iii) Continuation of Ongoing Research into Fatigue and Fracture of Metallic Materials:

Further analyses of fatigue crack growth in Mode I cyclic loading, when a single overload is introduced into an otherwise constant amplitude loading, have been conducted. Results have been obtained for several values of the ratio of the overload stress to the steady state cyclic stress. These results point to the interesting conclusion that, depending on the overload stress ratio, fatigue crack-growth can actually be stopped through the application of a single overload. Also, a thorough analysis of crack growth under pure Mode II cyclic loading was completed. These results point to a significant conclusion that fatigue cracks do not close under Mode II loading, unlike in the Mode I case.

Much progress has also been made in the analysis of fatigue behaviour of cracks near cold-worked fastener holes in plane-stress sheets.

The following is a cumulative list of publications (since 7/1/78) under this item of research under the present contract.

- (iii.1) M. Nakagaki and S. N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II," AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS Structures, Dynamics and Materials Conf., St. Louis, Missouri, April 1979, pp. 221-232.
- (iii.2) M. Nakagaki and S. N. Atluri, "Fatigue Crack-Closure and Delay Effects under Mode I Spectrum Loading," Proc. Third International Conference on Mech. Behaviour of Materials, Cambridge Univ., U.K., Pergamon Press, (to appear) 12 pages.
- (iii.3) S. N. Atluri, M. Nakagaki and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-Dimensional Problems of Engineering Fracture Mechanics," invited contribution, to appear in Finite Element Methods in Engineering Application (E. Krakeland, et al, Editors) Norwegian Institute of Technology, Norway 1979 (42 pages).

## PHASE R1

"Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

Introductory Remark

During this quarter, the work described below was performed. Emphasis has been heavily given to this work as a new vista has been revealed in bending theory.

Hygrothermal Effects

The doctoral thesis of R. P. Briley has been written, and the manuscript is being typed. This document thoroughly describes all of our pioneering beam vibration tests. A slightly modified version of the final thesis will be prepared for publication as an AFOSR technical report. This is the only appropriate media for dissemination of such a large volume of information.

At this juncture, we intend to survey the status of hygrothermal testing as it has developed over the last four years. We intend to coordinate our study with AFFDL research at Grumman Aerospace and the AMFL sponsored programs at General Dynamics - Fort Worth and Lockheed California Company. Mr. R. R. Valisetty has already begun this study. Our intention is to readily identify the future direction of our hygrothermal testing of resin matrix composites. Are there any glaring gaps or oversights in present and projected activities anywhere in the nation's composites effort?

New Bending Theory

Significant new theoretical developments have emerged from the work done with P. L. N. Murthy. The original intent of this work was to study the influence of hygrothermal effects on the design of stiffness critical composite structures; vibration and buckling behavior were to be empha-

sized. As mentioned in the last quarterly report, Mr. Murthy discovered some background theoretical problems that required treatment before the bulk of the study could proceed. This was a stroke of good fortune! It has lead to a significant breakthrough in engineering bending theory.

Hygrothermal effects in resin matrix/graphite composites manifest themselves by amplifying the importance of matrix controlled behavior modes. Transverse shear deformations, for example, play a greater role in the response to environmental loading. This effect was the first to be considered.

Shear deformation theory for homogeneous, isotropic beams originated in a paper by S. P. Timoshenko published in 1921. Since then, there have been some refinements and extensions to plates and shells, but no conceptual differences. Our recent reassessment of this theory has shown that there are two additional effects that (a) are the same order as transverse shear and (b) have never been included in any engineering bending theory. These effects are called non-classical bending and transverse normal strain effects.

To date, there is a complete static theory for the bending of homogeneous, isotropic beams. Predictions using this theory agree exactly with three-dimensional elasticity solutions for several distributed loading cases. (Beam axis response has been compared.) Currently, a dynamic extension of the theory is being sought. This is accompanied by a critical review of all of this work completed to date.

This is an exciting activity. Greater time and energy expended in this area is definitely warranted. The ultimate objective is a composite lamination theory that can then be applied to the hygrothermal response problems of original interest.

#### Dynamic Behavior of Woven Composites

In view of the breakthrough described above, this task has not been

pursued this quarter.

## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

Introductory Remarks

In this quarter, greater emphasis has been placed upon Phase R1. This is due to the fact that Mr. A. D. Reddy, the graduate research assistant primarily performing the isogrid related testing, was preoccupied with preparing for his doctoral qualifying examination. Never-the-less, considerable progress has been made.

After the decision not to proceed with building and testing circular cylindrical shells was made, the objectives of this research have been re-defined. There are two major new objectives. One is to acquire the much needed data base from the flat specimens already made and in hand. The other is to explore the general matter of using resistance-type strain gages on braided and woven structures. (This is, for example, a critical issue currently of importance in testing braided carbon-carbon rocket nozzles.)

A paper has been presented at the 20th AIAA SDM Conference in St. Louis. The buckling test work was included in this presentation.

Tests in Progress

Among the new element tests performed this quarter are tensile tests of rib elements ex situ and skin elements ex situ. The data to date is limited, but repeatable results have been obtained for the ribs. Problems associated with element tests of the braided skin have prompted the selection of the second objective mentioned previously. These ex situ tests complement in situ tests on isogrid beam elements to begin soon.

None of the above tests are intended to be destructive. Stiffness

information is all that is sought now. Additional strength tests will await a review of those data previously obtained.

The preliminary ex situ skin element tests performed have involved the use of resistance strain gages for sensing the mechanical response of the specimens. Because of the size and scale of the graphite yarns and the braid pattern, the indicated response (as sensed by the strain gages) is a strong function of gage placement. In order to smooth or average the measure of response, two approaches have been tried. One is to average the data obtained from separate gages. The other is to average the readings with the gage circuitry employed. There appears to be little significant difference between these approaches.

In addition to the above mechanical tests, preliminary fiber content tests have been completed on some rib and skin samples. A complete analysis of the test results was delayed due to a lack of information on the resin used. This information has been obtained, and the analysis is progressing.

#### Testing Needs

Ideas pertaining to testing of braided and woven composites are needed. This is prompted not only by our experience, but by that of McDonnell Douglas on braided carbon-carbon nozzles and combustors. This subject is being given serious thought.

New tests for determining shear deflection effects in grid-type structures are needed. An assessment of our previously conducted flexure tests will be made in order to help identify need candidate testing methods.

In general, our primary needs are to perform more element tests, to create a more substantial data base and to perform a critical analysis of the test methods used.



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FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report #4  
1 April 1979 - 30 June  
AFOSR Contract F49620-78-C-0085

L. W. Rehfield and S. N. Atluri  
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## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Engineering Science and Mechanics. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

Phase A

"Computational Methods for Fracture and Fatigue Analysis"

and

"Complementary Energy Approaches for Nonlinear Stability"

S. N. Atluri

Preface

The writer wishes to take this opportunity to bring to the attention of the AFOSR Program officials, certain items of interest concerning personnel actions at Georgia Tech. The appropriate administrators of the College of Engineering at Georgia Tech have recently considered the problem of research in computational methods in mechanics in general, and as to how to make the transfer of information, based on basic research, to the engineering development stage as efficiently as possible. The proposal has been made that Georgia Tech should organize, within the College of Engineering, a Center for the Advancement of Computational Mechanics. The aim of such a center would be to be a focal point not only for the development of computational methods for general purpose use in mechanics, but also for the development of computer systems (programs) that make an efficient use of these new methods. This writer's work, over the past six years, dealt mainly with the development of novel computational methods for fracture and fatigue analysis of metallic, and composite materials. Even though computer codes were developed in the course of this research, these codes are, in general, not of the variety that a potential user in industry/government laboratories can directly use. Based on interactions with several users in industry, and government laboratories, to whom computer codes, developed during the AFOSR Sponsored research, were released (with AFOSR concurrence) this writer feels the need to streamline the above codes so that they may be more efficient and more user-oriented. However, the writer's own interests do not extend in this direction. Thus the writer is very enthusiastic to become a member of the above mentioned Center at Georgia Tech. Also, becoming members in this center, as part of its Systems Laboratory, are several members of the faculty of Georgia Tech's School of Civil Engineering, whose primary interests are in the direction of computer systems/software of the type used in large-scale finite element analyses. Thus, the organization of the above center offers the writer the opportunity to collaborate with the other members in getting the research-oriented computer codes, developed under his AFOSR-sponsored research, streamlined so that they may be more user-oriented (these additional efforts, would not, however, be under AFOSR funding). In as much as the above center would, for purposes of administration, be located in the School of Civil Engineering at Georgia Tech, the appropriate administrators have decided that the writer should move to the School of Civil Engineering. Thus, as of 1 July 1979, the writer has become a member of the Structures Faculty of the School of Civil Engineering at Georgia Tech. The writer is very enthusiastic about this move, and the advantages it offers to conduct the present AFOSR-sponsored research more efficiently.

## Quarterly Progress Report

(4/1/79 - 6/30/79)

In the previous three months the following research was performed:

### i. Stress and Fracture Analysis of Angle-Ply Laminates

Development of a "multi-layer hybrid crack-element" for modelling cracks in angle-ply laminates has been completed, and the computer coding based on this has been verified to satisfaction. This new method makes it possible to calculate, directly, the mixed mode stress-intensity factors  $K_I$ ,  $K_{II}$ , and  $K_{III}$ , and their variation with the thickness coordinate,

near a crack which is oriented arbitrarily with respect to the material principal directions of each ply in an angle-ply laminate. Excellent results have been obtained for a  $(-45^\circ/+45^\circ/+45^\circ/-45^\circ)$  laminate containing edge and central cracks. These results have been prepared for publication.

Progress was made in the study of the problem of subcritical damage near the tip of a crack in an angle-ply laminate. Research was also conducted into the analysis of holes (and their size-effects) in angle-ply laminates. Specifically, the problem of a circular hole in  $(90/0/0/90)$  and  $(-45/+45/0/0/45/-45)$  laminates under uniform tension was studied. Based on these results, the possibility as well as desirability of developing a special "hole-element", which satisfies exactly the traction boundary conditions on the hole surface, is being explored. The problem of effects of free-edges on inter-laminar stresses has also been explored; and the feasibility of deriving a special "edge-element," for use in stress analysis of laminates, is being considered.

Modifications to the above procedures, to enable accurate analyses of cracks near fastener holes in adhesively-bonded metallic laminates, are being considered simultaneously.

The following is a cumulative list of publications (since 1 July 1978) and/or presentations under this item of research under the present contract:

- (i.1) "Cracks in Laminates" presentation, 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (i.2) T. Nishioka, and S.N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R.H. Sierakowski), University of Florida Press, pp. 95-101.
- (i.3) T. Nishioka and S.N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite-Element Approach - I" AIAA Paper No. 79-0801, Proc. 20th AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, St. Louis, Mo., April 1979, pp. 315-326.  
The above is also under review for publication in AIAA Journal.

- (i.4) S.N. Atluri, M. Nakagaki, and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws," International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
- (i.5) T. Nishioka and S.N. Atluri, "Analysis of a Through-Crack in  $(-45^\circ/+45^\circ/+45^\circ/-45^\circ)$  Laminate Using a Complementary Energy Principle," paper being submitted to Journal of Engineering Fracture Mechanics, August, 1979.
- (i.6) S.N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates," Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.
- (i.7) T. Nishioka, and S.N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates," being prepared for presentation and publication, 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Meeting, Seattle, Washington, 1980.
- (i.8) T. Nishioka and S.N. Atluri, "A New Approach to Estimate Free-Surface Effects for Surface Flaws," being prepared for submission to International Journal of Numerical Methods in Engineering.

#### ii. Two Dimensional Analyses of Heterogeneous Systems

Further results on two dimensional problems of cracks in lamina wherein the matrix and fibers are modeled individually have been obtained. Explorative studies into the feasibility of analysis of stable growth of a crack oriented at an angle to the fiber-direction and growing across the fiber, have been made.

All the stress-intensity factor solutions for two-dimensional heterogeneous systems are being systematically compiled for preparation of a report.

#### iii. Continuation of Ongoing Research into Fatigue and Fractures of Metallic Materials

Emphasis has been placed on the analysis of fatigue growth of cracks near fastener holes. Results have been obtained for sizes of plastic-zones near fastener holes subjected to various levels of cold-working. Excellent correlation of these results with the experimental results of Poolsuk and Sharpe (Journal of Applied Mechanics, December 1978) has been noted. Further, the present results were noted to agree with the experimental results better than most of the analytical estimates also cited by Poolsuk and Sharpe. Two cases, one with a crack lying entirely within the residual (due to cold-working) compressive stress zone, and the other with a crack of length greater than the radius of residual-plastic-zone due to cold-working, have been analysed in detail. Constant amplitude, zero-to-tension, fatigue loading was assumed. The significant conclusions from this study were: (a) the process of cold-working results in retardation of fatigue growth of cracks in both of the above cases; however, this retardation is higher in magnitude in the former case than in the latter; (b) in both the above cases, the retardation effects are more significant during the initial phase of fatigue crack-growth than after sufficient number of load-cycles.



Numerical results for crack-opening stresses and crack-closure stresses have been obtained in both the above cases. These results appear to provide an understanding of the fatigue-crack closure due to residual plasticity for cracks near cold-worked fastener holes.

The following is a cumulative list of publications (since 1 July 1978) under this item of research under the present contract.

- (iii.1) M. Nakagaki, and S.N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II" AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Mo., April 1979, pp. 221-232.  
This is also under review for publication in AIAA Journal.
- (iii.2) M. Nakagaki, and S.N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode-I Spectrum Loading," Proc. Third International Conference on Mechanical Behaviour of Materials, Cambridge University, United Kingdom, Pergamon Press, 12 p.  
The above is also scheduled to appear in the Journal of Fatigue, vol. 1, 1979.
- (iii.3) S.N. Atluri, M. Nakagaki, and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics," in Engineering Application of the Finite Element Method, vol. 1, Det Norske Veritas, Norway, 1979, pp. 14.1 - 14.32.
- (iii.4) S.N. Atluri, "Numerical Modeling of Crack Growth," Invited Contribution, to appear in Proc. 3rd ASCE, Engineering Mechanics Specialty Conference, Austin, Texas, September 16-19, 1979.
- (iii.5) S.N. Atluri, T. Nishioka, and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, By Moving Singular Elements," Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, to appear December, 1979, 32 pages.

#### iv. Complementary Energy Approaches for Nonlinear Stability

Basic exploratory work has been initiated to derive the beam, plate and shell equations starting with the a priori polar-decomposition of the deformation gradients into rigid-rotations and stretch (engineering strain). These equations, which are being developed in general curvilinear coordinates (and written in index-free notation), represent a novel departure in the technical theory of plates and shells. Since a significant number of nonlinear problems involve large rotations but small strains, the treatment of rigid-rotation directly as an independent variable in the kinematics, as well as in the complementary energy principle, would lead to significant improvements in the analysis of these problems. Analytical results generated so far, for a variety of beam stability problems, appears to confirm this view. After further exploratory analytical studies on uniqueness criteria etc., the above equations would be cast into finite element form to enable treatment of more complicated practical problems.

The following is a cumulative list of publications /presentations under this item of research under the present contract.

- (iv.1) S.N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis", Invited Contribution, International Conference on Theory and Practice of Finite Elements, Chalmers Institute of Technology, Gotenborg, Sweden, August 27-29, 1979.

## PHASE R1

### "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

#### Introductory Remark

During this quarter, the work described below was performed. Progress was retarded somewhat due to a lengthy illness suffered by Professor Rehfield during this time period.

#### Hygrothermal Effects

An effort is being made to further analyze the vibration data appearing in R. P. Briley's doctoral thesis. In particular, it is important to estimate the contributions to measured damping due to aerodynamic effects, moisture condensation and testing technique. This activity has been undertaken in response to suggestions received from AFML researchers who have a strong interest in this work.

An invited presentation was given at the USAF Workshop on Damping in Composite Structural Materials held at MIT on June 15. The above suggestions for further work were received at this meeting. Apparently our data remains the only dynamic data on hygrothermal effects in advanced resin matrix composites that exists.

A small-scale experimental study is to be launched in connection with the issue of sensitivity of damping measurements to testing technique. The unusual constraints imposed by hygrothermal environments result in compromises that are unnecessary in dry environments.

The survey of the status of hygrothermal testing practices has been expanded. With encouragement from Dr. C. C. Chamis of NASA Lewis Research Center, a survey paper will be prepared for presentation at the upcoming ASTM Conference on Composite Materials Testing and Design Allowables (October 1979, Dearborn, Michigan). Mr. R. R. Valisetty, a Graduate Research Assistant, will collaborate with Prof. Rehfield. Additional travel is planned for site visits and discussions with researchers in order to ensure completeness and lasting value of the study to workers in advanced composite structures.



Visits have been made already to Lockheed Palo Alto Research Laboratory and NASA Ames Research Center in connection with a trip to the recent ASTM Meeting in San Francisco.

#### New Bending Theory

Last quarter was a period of insight and discovery that produced a breakthrough in bending theory. This quarter is best described as a period of evaluation, scrutiny, verification and extension.

As reported in the last progress report, a complete static theory for the bending of homogeneous, isotropic beams was developed first. Predictions using this theory agree exactly with elasticity solutions for several distributed loading cases. The theory has been extended in two directions. A first approximation dynamic theory has been formulated. Also, a corresponding theory for orthotropic beams, a logical step in the direction of a lamination theory for composites, has been defined.

An evaluation of the first approximation dynamic theory for homogeneous, isotropic beams has been completed. Extremely fine agreement with the elasticity theory solution for a rectangular slab has been obtained. Comparison with the fully three-dimensional solution for a solid circular bar indicates some additional refinement for non-rectangular cross sections is needed. The nature of the discrepancy for circular cross sections is being investigated further.

Static and dynamic agreement of solutions obtained using the orthotropic theory with available elasticity solutions is quite good. The sensitivity of the dynamic theory to cross sectional shape has not arisen in this context due to the absence of elasticity solution benchmarks for orthotropic materials. A resolution for the isotropic material case, however, can be carried over to the orthotropic case in all likelihood.

All in all, this work is progressing well and should constitute a substantial contribution to structural theory.

## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

Introductory Remark

The work described below was performed during this quarter. Progress was retarded somewhat due to a lengthy illness suffered by Professor Rehfield during this time period.

Element Tests

Three types of element tests were performed. Short rib/skin specimens mounted in encapsulated ends have been supplied by McDonnell Douglas Astronautics Company. Fifteen specimens, five from each of the three parent isogrid panels, have been tested in simple compression. Extensional stiffness was monitored on six specimens fabricated with resistance strain gages installed. Ultimate compressive strength and an indication of the extent nonlinear material effects influence behavior were determined by testing to failure.

Ex situ tensile tests on rib elements were conducted. Specimens taken from ribs from all three parent panels have now been tested. These tests are for the determination of extensional stiffness rather than strength. The lack of uniformity of the elements produces experimental scatter. For this reason, additional specimens are being prepared to permit more testing. The intent of all of the ex situ testing is to provide elemental properties to be used in correlating theory with observed panel behavior.

Extensive damage occurred in the buckling tests of small panels cut from parent panel 3. In spite of this, it was possible to salvage several beam specimens of sufficient length to permit a series of three point bending tests to be performed. Panel 3 was the one of poorest quality. This is reflected in great geometrical variations in the specimens and large scatter in the stiffness and strength data taken in these tests. While correlation with theory may not be possible, careful documentation of the results will be taken.

Correlation and Analysis

The element compression tests provided data for correlation with theory. Measured rib and skin stiffness properties have been obtained

from ex situ element tests; these property values have been used to predict stiffnesses for the element compression specimens. The predictions compare quite favorably with averaged measured values obtained in the compression tests. This is in the face of experimental scatter due to variations in specimen quality. Consequently, extensional behavior appears to be reasonably well understood.

There are many unresolved issues surrounding bending behavior. The picture is clouded by the data scatter and by nonuniformity in the beam specimens. It appears that ultimate bending moment, for example, is a more consistent quantity than predicted ultimate stress values calculated on the basis of theory with measured geometric and property values. This is a frustrating situation.

All the three and four point beam bending tests strongly show a pronounced length effect that suggests transverse shear is important. This is consistent with the fact that a transverse shear correction helped to correlate the panel buckling data. A preliminary cantilever bending test series is underway with the intent of experimentally determining transverse shear stiffness. Further correlation of bending test data depends upon getting this matter satisfactorily resolved.

#### Closing Remarks

The efforts to understand and correlate data with theory are frustrated by two things. The specimens vary widely in quality due to the developmental nature of the manufacturing process for this type of structure. Also, the number of available specimens is small so that statistical methods, in view of the observed data scatter, are not reliable for such small sample sizes. Every bit of information and insight, however, will be gleaned from the specimens that can be. The structural concept is proven and should not be abandoned because the manufacturing process requires further development.

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FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report No. 5  
1 July 1979 - 30 September  
AFOSR Contract F49620-78-C-0085

L. W. Rehfield and S. N. Atluri  
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## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

Phase A

"Computational Methods for Fracture and Fatigue Analyses"

and

"Complementary Energy Approaches for Nonlinear Stability"

S. N. Atluri

In the previous three months the following research was performed:

(i) Stress and Fracture Analysis of Angle-Ply Laminates

Further analyses of symmetric angle-ply laminates, with through-the-thickness cracks, have been performed. Results were obtained not only for stress-intensity factors but also for details of distribution of transverse shear and normal stresses at the inter-laminar surfaces. These results provide a model for delamination and subcritical damage near a crack-tip. These results have been submitted for presentation at the forthcoming AIAA Structures Conference. Using the mixed-mode stress-intensity factors near the crack-tip in each ply, even under uniaxial loading symmetric with respect to the crack-axis, a model for non-self-similar crack growth has been constructed. In constructing this model, several crack-growth-direction criteria such as the maximum circumferential stress, minimum strain-energy density, and maximum strain energy release rate, have been used.

Preliminary results for stress-concentration factors around holes in angle-ply laminates have been obtained. Based on these results, it was felt that the development of a special hole-element may be more cost-effective. Accordingly, work has been initiated in this direction. In this approach, analytical asymptotic solutions for an elliptical hole in a general two-dimensional anisotropic solid subjected to far-field tension as well shear

are employed. From these two-dimensional solutions, a three-dimensional asymptotic solution, which accounts for the variation of the stress-concentration factors through the thickness of the lamina (and hence of the laminate), is generated by integration through the thickness. This procedure is analogous to the one successfully developed, in the previous months, for a crack which is, of course, the limiting case of an elliptical hole. Results from this phase of research are expected shortly.

Research has also been initiated into the development of special traction-free edge elements, as well as into the study of cracks near fastener holes in adhesively bonded metallic laminates.

The following is a cumulative list of publications (since 7/1/78) and or presentations under this item of research.

- (i.1) "Cracks in Laminates" presentation, 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (i.2) T. Nishioka and S. N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R. H. Sierakowshi), University of Florida Press, pp. 95-101.
- (i.3) T. Nishioka and S. N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite-Element Approach - I" AIAA Paper No. 79-0801, Proceedings 20th AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, St. Louis, Missouri, April 1979, pp. 315-326.  
The above is also accepted for publication in AIAA Journal.
- (i.4) S. N. Atluri, M. Nakagaki and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws", International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
- (i.5) T. Nishioka and S. N. Atluri, "Analysis of a Through-Crack in  $(-45^{\circ}/+45^{\circ}/+45^{\circ}/-45^{\circ})$  Laminate Using a Complementary Energy Principle", paper submitted to Journal of Engineering Fracture Mechanics, August 1979.
- (i.6) S. N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates", Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.



- (i.7) T. Nishioka and S. N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates", prepared for presentation and publication, 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Meeting, Seattle, Washington, 1980.
- (i.8) T. Nishioka and S. N. Atluri, "A New Approach to Estimate Free-Surface Effects for Surface Flaws", being prepared for submission to International Journal of Numerical Methods in Engineering.
- (i.9) S. N. Atluri and T. Nishioka, "Studies in Fracture of Composites", Contribution to U.S.-U.S.S.R. Conference on Mechanics of Composites, Lehigh University, September 1980.

(ii) Fatigue and Fracture of Metallic Materials

Studies have been completed on the closure-effects on fatigue-crack-propagation under Mode I and Mode II conditions and under general spectrum loading in center-cracked metallic panels.

Research is being planned for study of fatigue-crack-growth under mixed-mode (I and II) conditions, assuming conditions of small-scale-yielding near the crack-tip.

Some of the results are being presented (in absentia) in an invited paper at the U.S.-Japan Seminar on Elastic-Plastic Fracture.

The following is a cumulative list of publications (since July 1, 1978) under this item of research under the present contract.

- (ii.1) M. Nakagaki and S. N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II" AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Missouri, April 1979, pp. 221-232.  
This is also under review for publication in AIAA Journal.
- (ii.2) M. Nakagaki and S. N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode-I Spectrum Loading", Proceedings Third International Conference on Mechanical Behavior of Materials, Cambridge University, United Kingdom, Pergamon Press, 12 p.
- (ii.3) S. N. Atluri, M. Nakagaki and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics", in Engineering Application of the Finite Element Method, Vol. 1, Det Norske Veritas, Norway, 1979, pp. 14.1 - 14.32.  
The above is also accepted for publication in Computers and Structures.



- (ii.4) S. N. Atluri, "Numerical Modeling of Crack Growth", Invited Contribution, in Proceedings 3rd ASCE, Engineering Mechanics Specialty Conference, Austin, Texas, September 16-19, 1979.
- (ii.5) S. N. Atluri, T. Nishioka and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, By Moving Singular Elements", Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, to appear December 1979, 32 pages.
- (ii.6) A. S. Kobayashi, J. S. Cheng, S. N. Atluri, A. F. Emery and W. J. Love, "Elastic-Plastic Analyses of a Three-Point Bend Specimen and a Fracturing Pipe", invited contribution, U.S.-Japan Seminar on Elastic-Plastic Fracture, Hayama, Japan, November 1979.

(iii) Complementary Energy Approaches for Nonlinear Stability

By decomposing the deformation a priori into pure-stretch and rigid rotation, equations for plates undergoing large-deformations have been derived in terms of the Jaumann Stress measure and the rigid-rotation tensor of finite rotation. Uniqueness and Stability criteria for the solutions generated, based on a rational complementary energy principle (involving the Jaumann stress and rigid rotation tensors as variables) have been derived. A computer program, for finite element analysis of nonlinear buckling and post-buckling behavior of plates with various boundary conditions, is being developed, based on the above fundamentally novel developments.

The following is a cumulative list of publications/presentations under this item of research under the present contract.

- (iii.1) S. N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis", Invited Contribution, International Conference on Theory and Practice of Finite Elements, Chalmers Institute of Technology, Gotenborg, Sweden, August 27-29, 1979.
- (iii.2) S. N. Atluri, "Finite Strain Inelasticity, Complementary Energy, and Finite Elements: Some Recent Computational Studies", Invited Paper, U.S.-Europe Conference on Finite Elements in Nonlinear Structural Mechanics", Bochum, West Germany, July 1980.

## PHASE RI

### "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

#### Introductory Remarks

During this quarter, the work described below was performed. Progress was retarded somewhat due to a lengthy illness suffered by Professor Rehfield. Emphasis has been placed upon this phase, the new bending theory in particular.

#### Hygrothermal Effects

This effort has been bolstered by the arrival of Mr. Shlomo Putter from the Armament Development Authority, Israel Ministry of Defense. After only starting the first week in September, he has already begun a series of vibration experiments. He is on sabbatical and will be at Georgia Tech an entire year.

A new series of vibration experiments have been started which utilize electromagnetic noncontacting transducers and exciters. Flexural vibration tests at room temperature have been conducted for both aluminum and composite beams. These tests will further clarify the sensitivity of hygrothermal dynamic data to testing technique. Also, it will be possible to reduce parasite drag damping encountered in earlier tests by eliminating the cumbersome cable and accelerometer from the specimens. Damping values much less than those found by Briley are now being obtained.

The above activity will support our effort to refine the analysis of all of Briley's vibration data.

A presentation was made at the ASTM symposium held in Dearborn, MI. The complete reference is

Rehfield, L. W., and Valisetty, R. R., "A Survey of Hygrothermal Mechanical Test Methods for Graphite/Epoxy Composites", ASTM Symposium on Test Methods and Design Allowables for Fibrous Composites, Dearborn, MI, October 2-3, 1979.

The original intent was to have this survey complete and represent the combined wisdom of researchers throughout the country. Because of Professor Rehfield's illness, however, the study was not completed and a manuscript was not prepared. Essentially the presentation was a progress report rather than a report on the completed study.

### New Bending Theory

Our primary effort was expended on this area. Significant advances have been made.

One activity has been to generate new solutions to several example problems with the new theory and compile the results. This has been done in order to prepare for some planned future papers and presentations. Comparisons with Bernoulli-Euler and Timoshenko bending theories have been most enlightening. It appears that statically indeterminant applications provide the greatest differences; the new theory gives new information in such cases.

A study of fully three-dimensional effects is in progress using an exact elasticity solution as a baseline. This has lead to the discovery that there is a lateral strain effect that is of the same order as the other effects for some cross sections. A candidate theory accounting for all the essential physical effects has now been developed. It is being systematically evaluated at present.

Preliminary explorations related to a plate theory for homogeneous, isotropic materials and to a laminated beam theory for composites have been initiated. These efforts will accelerate when the three-dimensional effects referred to above are completely understood.

Closing Remarks

Two abstracts for papers and presentations have been prepared and sent for acceptance. The references are

Rehfield, L. W., "Toward a New Engineering Theory of Bending", AIAA/ASME 21st Structures, Structural Dynamics and Materials Conference, Seattle, WA, May 12-14, 1980.

Rehfield, L. W., "Nonlinear Elastic Vibration of Structures: Theory and Experiment", 22nd Israel Annual Conference on Aviation and Astronautics, March 12-13, 1980.

## PHASE R2

### "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

#### Introductory Remarks

The work described below was performed during this quarter. Progress was retarded somewhat due to a lengthy illness suffered by Professor Rehfield. Less emphasis was given to this phase during this period as the new bending theory developments and the arrival of Mr. Putter drew more attention. Never-the-less, a significant advance was recorded and will be described below.

#### Element Tests

Element testing continued with new beam and rib specimens being created by the Aerospace Shop. The new beam specimens have been geometrically characterized and tested in three- and four-point bending tests. In addition, ex situ rib specimens have been tested in tension. The increased number of tests should further our correlation efforts, in addition to contributing to our data base.

A complete series of new compressive buckling tests for beam specimens has been performed. A non-destructive stiffness plotting technique was used, so actual buckling was not permitted. The data obtained will be discussed in the next section. Two specimens, however, behaved in a rather peculiar manner. Stiffness plots of the usual type were not found and buckling loads could not be nondestructively estimated. A thorough examination revealed the presence of numerous cracks in the isogrid ribs of both beams. As a result, a repair technique was used and the specimens retested. Still no success! We tentatively conclude that the local cracks result in having the effect of strange geometric imperfections. At a later time, destructive buckling tests will be conducted on these two specimens.

#### Correlation and Analysis

If three- and four-point bending tests are performed on the same beam specimen, it is possible to estimate both the bending stiffness and transverse shear

stiffness from deflection data. This has been done for the new beam specimens. There is considerable variation in the stiffness values, even for beams cut from the same small panel. If, however, these stiffnesses are inserted in the column buckling equations, the resulting predictions are in excellent agreement with measured buckling loads. This is a significant advance!

Unfortunately, it has not been possible to quantitatively scale beam buckling data to panel buckling data. Constant ratios can be found from the data, but they are not understood at this time.

#### Closing Remarks

The correlation described above is most important and encouraging. This is particularly true in view of the scatter attributed to specimen nonuniformity.

An abstract has been prepared and sent for acceptance for presentation and publication. The complete reference is

Rehfield, L. W., and Reddy, A. D., "Behavior of Continuous Filament Advanced Composite Isogrid Structure", Third International Conference on Composite Materials, Paris, August 25-30, 1980.

C-14-614

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report No. 6  
1 October 1979 - 31 December  
AFOSR Contract F49620-78-C-0085

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## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.



Phase A

"Computational Methods for Fracture and Fatigue Analyses"

and

"Complementary Energy Approaches for Nonlinear Stability"

S. N. Atluri

In the previous three months (10/1/79 to 12/31/80) the following research was performed.

(I) Stress and Fracture Analysis of Angle-Ply Laminates

The development of special hole elements, based on an assumed stress hybrid approach, has been pursued and nearly completed. In this approach, analytical asymptotic solutions for an elliptical hole in a general two-dimensional anisotropic solid, subjected to far-field tensile stresses along the principal directions of material orthotropy as well as far-field shear stresses, are employed. From these two-dimensional solutions, a three-dimensional asymptotic solution which accounts for the variation of the stress-concentration factors through the thickness of the lamina (and hence of the laminate), and which accounts for the transverse shear and normal stresses, is generated by integration through the thickness of the laminate. This procedure is analogous to the one successfully developed earlier, in the course of the present contract, for through thickness cracks in laminates which are limiting cases of elliptical holes. The computer coding based on this approach is nearly debugged and preliminary results have been obtained. The obtained results for four ply and six ply symmetric angle-ply laminates agree excellently with those in prior literature and more over indicate the relative efficiency and accuracy of the presently developed procedure. These results are documented in paper accepted for presentation and publication at the forthcoming AIAA-SDM Conference in Seattle, May 1980.

Efforts have also been expended, in the present reporting period, on the development of special traction-free edge elements as well as the study of cracks near fastener holes in adhesively bonded metallic laminates.

The following is a cumulative list of publications (since July 1, 1978) and/or presentations under this item of research.

- (i.1) "Cracks in Laminates", Presentation 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (i.2) T. Nishioka and S. N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R. H. Sierakowshi), University of Florida Press, pp. 95-101.
- (i.3) T. Nishioka and S. N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite-Element Approach - I" AIAA Paper No. 79-0801, Proceedings 20th AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, St. Louis, Missouri, April 1979, pp. 315-326.  
The above is also accepted for publication in AIAA Journal.
- (i.4) S. N. Atluri, M. Nakagaki and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws", International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
- (i.5) T. Nishioka and S. N. Atluri, "Analysis of a Through-Crack in (-45°/+45°/+45°/-45°) Laminate Using a Complementary Energy Principle", paper submitted to Journal of Engineering Fracture Mechanics, August 1979.
- (i.6) S. N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates", Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.
- (i.7) T. Nishioka and S. N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates", accepted for presentation and publication, 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Meeting, Seattle, Washington, 1980.
- (i.8) T. Nishioka and S. N. Atluri, "A New Approach to Estimate Free-Surface Effects for Surface Flaws", being prepared for submission to International Journal of Numerical Methods in Engineering.
- (i.9) S. N. Atluri and T. Nishioka, "Studies in Fracture of Composites", Contribution to U.S.-U.S.S.R. Conference on Mechanics of Composites, Lehigh University, September 1980.
- (i.10) T. Nishioka and S. N. Atluri, "Efficient Finite Element Techniques for Computation of Mixed-Mode Stress Intensities for Cracks in Laminates", accepted for presentation and publication, 2nd International Conference on Numerical Methods in Fracture Mechanics, Swansea, United Kingdom, July 1980.
- (i.11) T. Nishioka and S. N. Atluri, "Studies on Fracture of Angle-Ply Laminates", accepted for presentation and publication, 4th International Conference on Fracture, Cannes, France, March 1981.

(ii) Fatigue and Fracture of Metallic Materials

Further results have been obtained for the effects of crack-closure on fatigue propagation of cracks near cold-worked fastener holes.

The method of moving singular elements developed for the above analysis have been found to be extremely novel in the analysis of more rapid crack propagation. Results obtained in this analysis are being presented in an invited paper to be presented at the 2<sup>nd</sup> International Conference on Numerical Methods in Fracture, Swansea, United Kingdom, July 1980.

Further, all the methods developed, under AFOSR support, for analysis of cracks in metallic materials are being summarized in an invited article to appear in a textbook on Finite Elements to be published by the American Society of Mechanical Engineers.

The following is a cumulative list of publications (since July 1, 1978) under this item of research under the present contract.

- (ii.1) M. Nakagaki and S. N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II", AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Missouri, April 1979, pp. 221-232.  
This is also accepted for publication in AIAA Journal.
- (ii.2) M. Nakagaki and S. N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode-I Spectrum Loading", Proceedings Third International Conference on Mechanical Behavior of Materials, Cambridge University, United Kingdom, Pergamon Press, 12 pages.
- (ii.3) S. N. Atluri, M. Nakagaki and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics", in Engineering Application of the Finite Element Method, Vol. 1, Det Norske Veritas, Norway, 1979, pp. 14.1 - 14.32.
- (ii.4) S. N. Atluri, "Numerical Modeling of Crack Growth", Invited Contribution, in Proceedings 3<sup>rd</sup> ASCE, Engineering Mechanics Specialty Conference, Austin, Texas, September 16-19, 1979.
- (ii.5) S. N. Atluri, T. Nishioka and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, By Moving Singular Elements", Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, to appear December 1979, 32 pages.
- (ii.6) A. S. Kobayashi, J. S. Cheng, S. N. Atluri, A. F. Emery and W. J. Love, "Elastic-Plastic Analyses of a Three-Point Bend Specimen and a Fracturing Pipe", invited contribution, U.S.-Japan Seminar on Elastic-Plastic Fracture, Hayama, Japan, November 1979.

- (ii.7) S. N. Atluri and T. Nishioka", Translating Singularity Method of Analysis of Rapid Crack Growth", invited paper, to appear in 2nd International Conference on Numerical Methods in Fracture, Swansea, United Kingdom, July 1980.
- (ii.8) S. N. Atluri, "Higher Order, Special, and Singular Elements", invited paper, Chapter 4 in Finite Element Methods, to be published by ASME, 1980.

(iii) Complementary Energy Approaches for Nonlinear Stability

A new derivation of the relevant field equations for plates and shells undergoing large rotations and deformations has been achieved: A new concept of a vector representation of finite rotations has been introduced. The incremental forms of these equations have been cast in terms of an incremental complementary energy principle with the symmetrized Biot/Lurel stress tensor and the rotations as variables.

Computer coding for beams undergoing large rotations and stretches has been completed and verified. Results have been obtained for post-buckling behaviour of beams under conservative as well as non-conservative loadings. Large-displacement solutions for beams subject to arbitrary loading have also been obtained. These results effectively serve to bring out the salient features of the present fundamentally novel analysis procedure. These results are being presented in an invited paper at the International Conference on Nonlinear Mechanics and Finite Elements, Washington, DC, October 1980.

The following is a cumulative list of publications/presentations under this item of research under the present contract.

- (iii.1) S. N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis", Invited Contribution, International Conference on Theory and Practice of Finite Elements, Chalmers Institute of Technology, Gotenborg, Sweden, August 27-29, 1979.
- (iii.2) S. N. Atluri, "Finite Strain Inelasticity, Complementary Energy, and Finite Elements: Some Recent Computational Studies", Invited Paper, U.S.-Europe Conference on Finite Elements in Nonlinear Structural Mechanics, Bochum, West Germany, July 1980.
- (iii.3) S. N. Atluri, H. Murakawa, and K. W. Reed, "Stability Analysis Via a New Complementary Energy Principle" to be presented at 2nd International Conference on Nonlinear Mechanics and Finite Elements, JIAFS and GWU, Washington, DC, October 1980.



## PHASE RI

### "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

#### Introductory Remarks

During this quarter, the work described below was performed. Progress was retarded somewhat due to health problems suffered by Professor Rehfield.

#### Hygrothermal Effects

New vibration experiments have been performed which utilize electro-magnetic noncontacting transducers and exciters. Both aluminum and composite beams have been tested at room temperature and elevated temperature. The primary purpose of these tests is to permit more quantitative, useful information to be extracted from the data generated earlier in Briley's experiments.

The contribution of aerodynamic damping to the damping measured by Briley is of considerable practical importance. The usual approach is to test specimens in a vacuum chamber, thereby eliminating aerodynamics altogether. This approach was not used because near in situ hygrothermal environmental tests were required. For small amplitudes, aerodynamic damping is proportional to amplitude. Therefore, tests have been conducted at several amplitudes. Damping vs. amplitude plots are constructed. Extrapolation to zero amplitude yields intrinsic material damping and aerodynamic damping. This technique is quite effective, and, since the test conditions and setup are the same as those used by Briley, good estimates of the aerodynamic damping contributions to the previous results have been obtained. Material and air damping values are the proper magnitude and agree well with values published in the literature.

The other extraneous contribution to damping in Briley's tests is due to parasite effects associated with the accelerometer cable. These effects may be partially attributable to aerodynamics and partially due to mechanical deformation of the cable. Regardless of the cause, a reliable estimate of this contribution is

desirable. Tests intended to determine the cable contribution are under way. Preliminary results suggest that the cable damping is largely independent of amplitude. If this is true, simple adjustments to Briley's data can be made.

### New Bending Theory

Consistent progress has been made in developing and applying the new bending theory. Emphasis on the application to beams has been given to flexural vibrations. Both isotropic and orthotropic beams have been considered for the following four limiting cases of boundary restraint:

- (1) fully clamped
- (2) clamped-propped
- (3) simply supported
- (4) cantilever

Comparisons with Bernoulli-Euler and Timoshenko theories have been made. A number of observations have been made:

- (1) natural frequencies and mode shapes predicted by the new theory are close to Timoshenko theory predictions;
- (2) bending moment values for fully clamped and clamped-propped beams depart significantly from Timoshenko theory predictions;
- (3) differences in predictions between our new theory and Timoshenko theory are more pronounced for orthotropic beams;
- (4) the additional factors accounted for in the new theory appear to be more significant for static applications.

One of the significant hygrothermal effects on resin matrix composites is the reduction in resin controlled stiffness properties. A preliminary assessment of the effect of hygrothermally induced transverse shear stiffness reduction has been made for unidirectionally laid up beams (which are orthotropic). Properties corresponding to three environmental conditions were used. Our findings suggest very significant changes in behavior at the higher moisture content levels. This

finding was suspected all along, and a study of this type served as a primary incentive for working on a new theory. Thus, the importance and significance of this work has now been amply demonstrated.

Additional work on three dimensional beam effects and laminated beams was not pursued this quarter. Exploratory work on a new plate theory was assigned a higher priority.

The extension of the ideas behind the new bending theory to plates was explored. A new plate bending for isotropic plates has been developed. Several limited applications have been considered which facilitate comparisons with Reissner's plate equations. Thus far, findings have been analogous to those for the beam theory. This work is quite interesting and encouraging and has both fundamental and applied consequences.

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## PHASE R2

### "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

#### Introductory Remarks

The work described below was performed during this quarter. Progress was retarded somewhat due to health problems suffered by Professor Rehfield.

#### Element Tests

The three original large isogrid panels were cut up to produce four small panels each. Half of the small panels, after having been nondestructively compression tested, were cut to produce two beams each. The beams have been tested extensively, and the results described in earlier progress reports. This quarter the remaining small panels (five in number, as one was damaged during the early compression tests) have been tested in bending in an analogous manner to the beams.

Three- and four-point bending tests were conducted on the small panels. Since the panels are wider than the beams tested earlier, a new testing setup was required and the fixture required modification. In addition, the early tests yielded non-repeatable stiffness values. Good repeatability was obtained after machining the ends of specimens flat to prevent rocking on the supports. Both skin up and skin down tests were performed.

As mentioned in the previous progress report, three- and four-point test data taken together permit both the bending and transverse shear stiffnesses to be estimated. These estimates were obtained. Their use in correlation and analysis will be discussed in the next section.

Erratic behavior during buckling tests lead to the discovery of damage, specifically cracks in the ribs, in three beams. A repair technique was used which consisted of injecting a contact adhesive into the cracks. Bending tests were repeated after repair. The results are in good agreement with those obtained prior to the discovery of the damage.



Tests were performed on a skin specimen to determine Poisson's ratio. It is 0.42. 0.33 was the value assumed in the design analysis.

### Correlation and Analysis

The bending tests of the small panels yielded estimates for bending and transverse shear stiffnesses. These stiffness values were then used to predict compressive buckling loads from a smeared stiffener theory. The theoretical predictions obtained in this way agree quite well with the experimentally determined buckling loads for the same panels found earlier. This is an important finding in our efforts to correlate theory with experiment. It is particularly significant in view of the panel-to-panel variability (and concomitant experimental scatter) due to manufacturing inconsistencies. Similar correlation of theoretical predictions based upon measured stiffnesses with buckling experimental results was reported previously for beam specimens.

Scaling is an extremely important issue. It is very desirable to be able to interpret and relate experiments on beams, small panels and large panels. There are two reasons for this. First, the use and applicability of simple smeared stiffener theory depends on reliable scaling. Second, beam tests are simple, easy and relatively inexpensive to perform; whenever possible, it is desirable to test beams rather than larger, more complex structures. This quarter, the scaling issue has been largely resolved. Measured stiffnesses on small panels have been used successfully to predict buckling loads of the large panels. In addition, even though smeared stiffener theory does not apply to beams which consist of a single stiffener, an approximate scaling law based upon elementary physical reasoning has been developed for relating beams to small panels. This completes the necessary basis for stiffness data interpretation.

E-16-612

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report No. 7  
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AFOSR Contract F49620-78-C-0085

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## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

## Phase A.

"Computational Methods for Fracture and Fatigue Analyses"  
and  
"Complementary Energy Approaches for Nonlinear Stability".

S.N. Atluri

In the previous three months (1/1/80 to 3/31/80) the following research was accomplished.

(I) Stress and Fracture Analysis of Angle-ply Laminates

The development of special laminated "hole-elements", for failure analysis near fastener holes in angle-ply laminates, has been completed. The development of these special elements, in which analytical asymptotic solutions are embedded, is based on a complementary energy principle with subsidiary constraints. The details of the developed method are as follows: (i) each layer of the laminate is treated as an anisotropic medium and the material properties of the laminate in global coordinates vary from layer to layer; (ii) the three-dimensional stress state, including the interlayer normal and shear stresses, is accounted for; (iii) the analytical asymptotic solution near the hole is embedded in the special elements; (iv) the assumed equilibrated stress-solution in the laminate is such that the interlayer traction reciprocity condition is satisfied a priori, but the planar stresses in the plane of the laminate are allowed to be discontinuous at the interlayer interface; (v) each finite element consists of the entire series of layers in the laminate; (vi) the inter-element traction reciprocity is enforced, a posteriori, in the variational formulation, through a Lagrange multiplier technique, and (vii) the procedures developed herein lead to a direct evaluation stress-concentration factors, and their variation from lamina to lamina, near the hole.

The above procedures for analysing holes, in conjunction with the analysis procedures for cracks developed earlier under the present research contract, represent the most efficient techniques for failure analysis of angle-ply laminates reported so far in the literature.

Detailed results have been obtained in a few carefully chosen test cases such as fastener holes in 4-ply (0/90)s and ( $\pm 45$ /s) laminates.

Two papers based on the above developments for holes have been prepared and will be presented at the forthcoming (i) 21st AIAA Structures, Structural Dynamics and Materials Conference in Seattle, May 1980, and (ii) 4th International Congress on Fracture, Cannes, France, March 1981.

The following is a cumulative list of publications (since 7/1/78) and/or presentations under this item of research.

- (i.1) "Cracks in Laminates", Presentation 4th USAF/Army?Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (i.2) T. Nishioka and S.N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R.H. Sierakowshi), University of Florida Press, pp. 95-101.
- (1.3) T. Nishioka and S.N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite-

Element Approach - I" AIAA Paper No. 79-0801, Proceedings 20th AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, St. Louis, Missouri, April 1979, pp. 315-326.

- (i.4) S.N. Atluri, M. Nakagaki and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws", International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
  - (i.5) T. Nishioka and S.N. Atluri, "Analysis of a Through-Crack in  $(-45^{\circ}/+45^{\circ}/+45^{\circ}/-45^{\circ})$  Laminate Using a Complementary Energy Principle", paper submitted to Journal of Engineering Fracture Mechanics, August 1979.
  - (i.6) S.N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates", Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.
  - (i.7) T. Nishioka and S.N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates", accepted for presentation and publication, 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Meeting, Seattle, Washington, 1980.
  - (i.8) T. Nishioka and S.N. Atluri, "A New Approach to Estimate Free-Surface Effects for Surface Flaws", being prepared for submission to International Journal of Numerical Methods in Engineering.
  - (i.9) S.N. Atluri and T. Nishioka, "Studies in Fracture of Composites", Contribution to U.S.-U.S.S.R. Conference on Mechanics of Composites, Lehigh University, September 1980.
  - (i.10) T. Nishioka and S.N. Atluri, "Efficient Finite Element Techniques for Computation of Mixed-Mode Stress Intensities for Cracks in Laminates", accepted for presentation and publication, 2nd International Conference on Numerical Methods in Fracture Mechanics, Swansea, United Kingdom, July 1980.
  - (i.11) T. Nishioka and S.N. Atluri "Studies on Fracture of Angle-Ply Laminates", accepted for presentation and publication, 4th International Conference on Fracture, Cannes, France, March 1981.
  - (i.12) T. Nishioka and S.N. Atluri, "Assumed Stress Finite Element Analysis of Through-Cracks in Angle-Ply Laminates", AIAA Journal, to appear, September 1980, 45 pages.
- (II) Fatigue and Fracture of Metallic Materials

The study of effects of fatigue crack-closure on the propagation of cracks near cold-worked fastener holes under cyclic loading has been completed.

The method of moving singular elements developed for the above analysis has been found to be extremely useful in a novel analysis of fast crack propagation. Results obtained in this analysis have been prepared as a paper to be presented at the 2nd International Conference on Numerical Methods in Fracture, Swansea, U.K., July 1980.

All the methods for analysing singularities, developed by the writer and his students under AFOSR support, have been summarized in the form of a chapter in a text book on Finite Element Methods, upon invitation from the committee on Computing in Applied Mechanics of ASME.

The following is a cumulative list of publications (since 7/1/78) under this item of research under the present contract.



- (ii.1) M. Nakagaki and S.N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II", AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Missouri, April 1979, pp. 221-232.  
This is also accepted for publication in AIAA Journal.
- (ii.2) M. Nakagaki and S.N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode-I Spectrum Loading", Proceedings Third International Conference on Mechanical Behavior of Materials, Cambridge University, United Kingdom, Pergamon Press, 12 pages.  
The above has also appeared as an invited paper in the Journal, Fatigue of Engineering Materials & Structure, Pergamon, Vol. 1, No. 4, 1979, pp 474-484.
- (ii.3) S.N. Atluri, M. Nakagaki and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics", in Engineering Application of the Finite Element Method, Vol. 1, Det Norske Veritas, Norway, 1979, pp 14.1-14.32.
- (ii.4) S.N. Atluri, "Numerical Modeling of Crack Growth", Invited Contribution, in Proceedings 3rd ASCE, Engineering Mechanics Specialty Conference, Austin, Texas, September 16-19, 1979.
- (ii.5) S.N. Atluri, T. Nishioka and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, By Moving Singular Elements", Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, to appear December 1979, 32 pages.
- (ii.6) A.S. Kobayashi, J.S. Cheng, S.N. Atluri, A.F. Emery and W.J. Love, "Elastic-Plastic Analyses of a Three-Point Bend Specimen and a Fracturing Pipe", invited contribution, U.S.-Japan Seminar on Elastic-Plastic Fracture, Hayama, Japan, November 1979.
- (ii.7) T. Nishioka, D.B. Stonesifer, and S.N. Atluri, "Translating Singularity Method of Analysis of Rapid Crack Growth", invited paper, 2nd International Conference on Numerical Methods in Fracture, Swansea, U.K., July, 1980.
- (ii.8) S.N. Atluri, Higher-order, Special and Singular Elements, Invited Paper, Chapter 4 in Finite Element Methods, published by ASME, NY, (to appear) 1980.
- (III) Complementary Energy Approaches for Nonlinear Stability

Using the polar decomposition theorem to split the deformation into pure stretch and rigid rotation, the field equations governing arbitrarily large deformations of beams, plates, and shells have been newly derived. This represents a fundamental departure in the technical theories of these structural members.

Based on the above equations, complementary energy principles, involving the symmetrized Biot Stress, and rigid rotation as variables, have been derived. Incremental forms of these principles, in both total as well as updated Lagrangean forms have been derived.

Results have been obtained for post-buckling behavior of beams and plates. The problem of the classical elastic has also been solved and the results were found to agree excellently with the celebrated results of S. Timoshenko. These results have been prepared as a paper for publication in the Journal of Computers and Structures, and will be presented at the International Conference on Nonlinear Mechanics and Finite Elements, Washington, DC, October, 1980.

The following is a cumulative list of publications/presentations under this item of research under the present contract.

- (iii.1) S.N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis", Invited Contribution, International Conference on Theory and Practice of Finite Elements, Chalmers Institute of Technology, Gottenborg, Sweden, August 27-29, 1979.
- (iii.2) S.N. Atluri, "Finite Strain Inelasticity, Complementary Energy, and Finite Elements: Some Recent Computational Studies", Invited Paper, U.S.-Europe Conference on Finite Elements in Nonlinear Structural Mechanics, Bochum, West Germany, July 1980.
- (iii.3) S.N. Atluri, H. Murakawa, and K.W. Reed, "Stability Analysis Via a New Complementary Energy Principle" to be presented at 2nd International Conference on Nonlinear Mechanics and Finite Elements, JIAFS and GWU, Washington, DC, October 1980. (Also to appear in Journal of Computers & Structures, Pergamon).

## PHASE R1

### "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

#### Introductory Remarks

During this quarter, the work described below was performed. Progress in the area of hygrothermal vibration was substantial. An important development in connection with the new bending theory was achieved as well.

#### Hygrothermal Effects

A major addition to our vibration testing capability was added this quarter--- a vacuum test chamber. This chamber is for room temperature experiments. The vacuum environment eliminates all aerodynamic damping from consideration. It was designed, built and utilized in both pilot, shakedown experiments and research related tests. It seems to perform admirably.

Damping-frequency experiments have been performed on composite specimens using the electro-magnetic noncontacting transducers and exciters and the new vacuum chamber. The specimens were dry and at room temperature. The tests are resonance tests. The natural frequency is varied by use of three approaches: mass addition, excitation of higher modes, and variation of unsupported beam length. Data on behavior in the 0-1000 Hertz range has been obtained.

Estimates of parasite effects due to the accelerometer cable have been established within reasonable limits for the early tests by Briley. Cable-related damping appears to be frequency dependent. Consequently, corrections for each type of composite beam specimen have been determined by controlled tests with and without a mounted accelerometer. This permits Briley's data to be translated into a more useful form.

#### New Bending Theory

A major development was achieved in the theory. A three-dimensional vari-



ation of the displacement field through the depth of the beam or plate has been determined in terms of the reference axis or surface displacement field. This had been absent and prevented further development of a dynamic theory.

With the exception of the above, effort was directed toward synthesis, organization and reporting activities. Also, an extensive historical review has been conducted to assure that all earlier contributions are duly noted.

## PHASE R2

### "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

#### Introductory Remarks

The work described below was performed during this quarter. Considerable progress was made. All the research work originally proposed has now been completed. Synthesis, organization and reporting remains. A new task, the effect of damage on compressive buckling behavior, will be undertaken as a final experimental effort.

#### Element Tests

Short beam rib elements were prepared from remnants taken from all three parent panels. Short beam three-point bending tests were conducted to determine interlaminar shear strength. The shear failures occurred at depths that correspond approximately to maximum shear stress according to theory. While the data showed scatter (a standard deviation of 19 percent of the mean value), the mean ultimate interlaminar shear stress compares favorably with the value assumed in the design analysis. The former is 14,245 psi, the latter 14,000 psi. This is obviously encouraging and confirms that the behavior is predictable.

#### Correlation and Analysis

Two types of strength tests had been conducted earlier. One is for determining ultimate compressive strength. The encapsulated specimens prepared by McDonnell Douglas were for this purpose. The second were three- and four-point bending tests conducted upon beam specimens. The strength data from all these tests have been studied this quarter.

Correlation of compressive strength data was based upon the values assumed for ultimate stresses in the design analysis for ribs and skin. Estimates from micro-mechanics were used to verify that these values are reasonable. The total com-

pressive load is shared by skin and rib. Failure was assumed to occur if the load contribution in either was greater than the ultimate value calculated using design ultimates and measured dimensions. In the vast majority of cases encountered, this elementary approach identified the failure correctly as being in the rib, skin or both. In addition, quantitative predictions provide fair-to-good correlation with the measured values.

In view of the pilot nature of the hardware, this is a very satisfactory situation. Compressive behavior is now understood.

The situation is not as satisfactory for bending strength. The data are limited, but poor correlation of ultimate stresses calculated from test data and design ultimates was generally found. Some is to be expected because ribs are far from rectangular and uniform and bending strength is generally less than uniaxial strength. Fifty percent or so is a typical discrepancy, however. Surprisingly, considerable consistency in ultimate bending moments was found. Ultimate moments from three-point tests tended to be about 10 percent less than those from four-point tests. This suggests that the failure process is influenced to some extent by a combination of bending and transverse shear stresses.

All of the failures can be described as bending failures. The failure sites were located in the ribs, on the free surface side and in the neighborhood of grid node points. It appears that complex, three-dimensional stress states are responsible. Calculations indicate that the shear stresses according to elementary theory are well below those required to produce interlaminar shear-type failures. It is not surprising, therefore, that ultimate bending stress calculations based upon elementary theory provide poor predictions in view of the failure sites.

Purely theoretical buckling predictions for large and small panels have been made using measured rib fiber volume fractions and the rule of mixtures, measured skin modulus and measured geometric dimensions. Elementary smeared isogrid

theory, modified to include the necessary effect of transverse shear deformation, was the basis. Three ways of estimating transverse shear stiffness were used and compared. Correlation with experimental data is excellent in view of the quality and nonuniformity of the hardware. The use of a simple average transverse shear stress to estimate shear stiffness appears to give the best correlation. This study concludes our buckling correlation work. All issues are resolved and the degree of correlation is most satisfying.

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report No. 8  
1 April 1980 - 30 June  
AFOSR Contract F49620-78-C-0085

L. W. Rehfield and S. N. Atluri  
College of Engineering  
Georgia Institute of Technology  
Atlanta, Georgia 30332

## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

# Phase A

## "Computational Methods for Fracture and Fatigue Analyses" and "Complementary Energy Approaches for Nonlinear Stability"

S.N. Atluri

In the previous three months (4/1/80 to 6/30/80) the following research was accomplished.

### I. Stress and Fracture Analysis of Angle-Ply Laminates:

Special laminated "hole elements" for failure analysis of fastener holes in laminated plates, the development of which was reported in the previous progress report (1/1/80 to 3/31/80), was extensively tested. Detailed results have been obtained for 3-dimensional stress states near holes in several cases of angle-ply laminated. These results were found to be radically different from those published in literature by other investigators. An extensive study of the reasons for these discrepancies was conducted, and confidence in the correctness of the present results has been established. From a close examination and understanding of the present 3-dimensional analysis results, simple approximate 2-dimensional analysis models for through-thickness holes in angle-ply laminates (with any number of lamina) have been developed.

Two papers based on the above results, one for publication in AIAA Journal and the other for publication in Jnl. of Composite Materials, are being prepared.

The following is a cumulative list of publications and/or presentations under this item of research.

- (i.1) "Cracks in Laminates", Presentation 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (i.2) T. Nishioka and S.N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R.H. Sierakowshi), University of Florida Press, pp. 95-101.

- (i.3) T. Nishioka and S.N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite-Element Approach - I" AIAA Paper No. 79-0801, Proceedings 20th AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, St. Louis, Missouri, April 1979, pp. 315-326.
- (i.4) S.N. Atluri, M. Nakagaki and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws", International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
- (i.5) T. Nishioka and S.N. Atluri, "Analysis of a Through-Crack in  $(-45^{\circ}/+45^{\circ}/+45^{\circ}/-45^{\circ})$  Laminate Using a Complementary Energy Principle", paper submitted to Journal of Engineering Fracture Mechanics, August 1979.
- (i.6) S.N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates", Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.
- (i.7) T. Nishioka, and S.N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates" 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, Seattle, Wash., May 1980, pp. 295-302.
- (i.8) T. Nishioka and S.N. Atluri, "A New Approach to Estimate Free-Surface Effects for Surface Flaws", prepared for submission to International Journal of Numerical Methods in Engineering.
- (i.9) S.N. Atluri and T. Nishioka, "Studies in Fracture of Composites", Contribution to U.S.-U.S.S.R. Conference on Mechanics of Composites, Lehigh University, September 1980.
- (i.10) T. Nishioka and S.N. Atluri, "Efficient Finite Element Techniques for Computation of Mixed-Mode Stress Intensities for Cracks in Laminates", presented and published at, 2nd International Conference on Numerical Methods in Fracture Mechanics, Swansea, United Kingdom, July 1980.
- (i.11) T. Nishioka and S.N. Atluri, "Studies on Fracture of Angle-Ply Laminates", accepted for presentation and publication, 4th International Conference on Fracture, Cannes, France, March 1981.
- (i.12) T. Nishioka and S.N. Atluri, "Assumed Stress Finite Element Analysis of Through-Cracks in Angle-Ply Laminates", AIAA Journal, to appear, September 1980, 45 pages.
- (i.13) T. Nishioka, and S.N. Atluri, "Analysis of Through-holes in Composite Laminates: A Special Hole Element Approach" being submitted to AIAA Journal
- (i.14) T. Nishioka, and S.N. Atluri, "An Approximate 2-D Analysis of Stress Concentration Around Holes in Laminates", being submitted to Journal of Composite Materials.



## II. Fatigue and Fracture of Metallic Materials:

In the current period, attention was primarily focussed on analysis of cracks in one of the layers of an adhesively bonded metallic laminate. These problems were studied for both the cases of debonding and no debonding of the lamina in the vicinity of the crack. Parametric studies with variations of crack-length, debond-area, and material properties of the lamina, were conducted. These results are being currently summarized for publication.

The following is a cumulative list of publications (since 7/1/78) under this item of research under the present contract.

- (ii.1) M. Nakagaki and S.N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II", AIAA Paper No. 1 79-0758, 20th AIAA/ASME ASCE/AHS DSM Conference, St. Louis, Missouri, April 1979, pp. 221-232.

This is also accepted for publication in AIAA Journal.

- (ii.2) M. Nakagaki and S.N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode-I Spectrum Loading", Proceedings Third International Conference on Mechanical Behavior of Materials, Cambridge University, United Kingdom, Pergamon Press, 12 pages.

The above has also appeared as an invited paper in the Journal, Fatigue of Engineering Materials & Structure, Pergamon, Vol. 14, 1979, pp. 474-484.

- (ii.3) S.N. Atluri, M. Nakagaki and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics", in Engineering Application of the Finite Element Method, Vol. 1, Det Norske Veritas, Norway, 1979, pp. 14.1-14.32.

Also to appear in Journal of Computers and Structures, 1980.

- (ii.4) S.N. Atluri, "Numerical Modeling of Crack Growth", Invited Contribution, in Proceedings, 3rd ASCE, Engineering Mechanics Specialty Conference, Austin, Texas, September 16-19, 1979.

- (ii.5) S.N. Atluri, T. Nishioka and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, By Moving Singular Elements", Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, AMD Vol. 25, Pp. 66-102.

- (ii.6) A.S. Kobayashi, J.S. Cheng, S.N. Atluri, A.F. Emery and W.J. Love, "Elastic-Plastic Analyses of a Three-Point Bend Specimen and a Fracturing Pipe", invited contribution, U.S.-Japan Seminar and Elastic-Plastic Fracture, Hyama, Japan, November 1979.

- (ii.7) T. Nishioka, R.B. Stonesifer, and S.N. Atluri, "Translating Singularity Method of Analysis of Rapid Crack Growth", invited paper, 2nd International Conference on Numerical Methods in Fracture, Swansea, U.K., July 1980.
- (ii.8) S.N. Atluri, Higher-order, Special and Singular Elements, Invited Paper, Chapter 4 in Finite Element Methods, published by ASME, NY, (to appear) 1980.
- (ii.9) T. Nishioka, and S.N. Atluri, "Analysis of Cracks in Adhesively Bonded Metallic Laminates", prepared for presentation and publication at 22nd AIAA/ASME/ASCE/AHS SDM Conference, Atlanta, April 1981.

### III. Complementary Energy Approaches for Nonlinear Stability:

Using the newly developed field equations in terms of pure stretches, rigid rotations, and Jaumann Stresses, as reported in the previous report (1/1/80 to 3/31/80), finite element models have been developed for analysis large-deformation and post-buckling behavior of plates and shallow shells. The results obtained in this investigation are being prepared for publication.

The following is a cumulative list of publications/presentations under this item of research under the present contract.

- (iii.1) S.N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis", Invited Contribution, International Conference on Theory Practice of Finite Elements, Chalmers Institute of Technology, Gotenborg, Sweden, August 27-29, 1979.
- (iii.2) S.N. Atluri, "Finite Strain Inelasticity, Complementary Energy, and Finite Elements: Some Recent Computational Studies", Invited Paper, U.S.-Europe Conference on Finite Elements in Nonlinear Structural Mechanics, Bochum, West Germany, July 1980.
- (iii.3) S.N. Atluri, H. Murakawa, and K.W. Reed, "Stability Analysis Via a New Complementary Energy Principle" to be presented at 2nd International Conference on Nonlinear Mechanics and Finite Elements, JIAFS and GWU, Washington, DC, October 1980. (Also to appear in Journal of Computers & Structures, Pergamon).
- (iii.4) H. Murakawa, and S.N. Atluri, "Complementary Energy Analysis of Large Deformations and Post-Buckling of Plates" being prepared for presentation and publication at 22nd AIAA/ASME/ASCE/AHS SDM Conference, Atlanta, April 1981.

## PHASE RI

## "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

Introductory Remarks

During this quarter, the work described below was performed. Progress in all areas was quite good. A comparative study of the dynamic behavior of woven composites and unidirectional tape composites was completed. The results indicate hygrothermal testing is not warranted for the woven composites as the results do not differ substantially from the unidirectional tape specimens.

Hygrothermal Effects

A major addition to our vibration testing capability was added this quarter --- the Hewlett Packard 5423 Structural Dynamics Analyzer. This is a Fourier-type analyzer with hardwired capability to determine damping and other response features. Currently studies are being conducted to get acquainted with the system. Comparisons with damping measurements performed our usual manual way are being performed at present.

Damping-frequency experiments have been performed on composite specimens in hygrothermal environments. Two types of specimens have been tested --- type A ((0) ply layup) and type B ([+45] ply layup). Both are made from Narmco 5208/300 graphite/epoxy. Type A exhibits fiber controlled behavior in bending. By contrast, type B behavior is matrix controlled. The tests are resonance tests. The natural frequency is varied by use of three approaches: mass addition, excitation of higher modes, and variation of unsupported beam length. Data in the 10-1000 Hertz range has been obtained for saturated cantilever beam specimens at 180°F and 140°F.

The results of the tests are currently being studied. Discrete points agree reasonably well with Briley's earlier tests.

An abstract was prepared, submitted and accepted for a paper to be presented at the ASTM Symposium on Composites for Extreme Environments. The complete reference is

Rehfield, L. W., Briley, R. P., Putter, S., "Dynamic Tests of Graphite/Epoxy Composites in Hygrothermal Environments," ASTM Symposium on Composites for Extreme Environments, Bal Harbor, FL, Nov. 11-12, 1980.

### New Bending Theory

Much of the work on beams has been concerned with preparations for publication. An extensive literature survey on bending theories is near completion. Also, drawings, figures and tables have been prepared for slides and photographs.

Additional work has been devoted toward extending the underlying ideas of our bending theory to plates. In particular, a careful evaluation of earlier work by Reissner is in progress. Comparisons are being made among various solutions to particular plate problems. It is felt that type of study will clarify the relationships among competing engineering theories.

### Dynamic Behavior of Woven Composites

Room temperature tests have been performed in our vacuum chamber on two types of woven graphite/epoxy beam specimens. Also, similar tests have been performed on (0) and (45) beam specimens of the same materials system but of unidirectional tape prepreg. The testing procedure used is identical to our earlier tests.

As mentioned in the introductory remarks, woven cloth and unidirectional tape composites behave in the same manner. The similarity in damping characteristics is a significant finding.

The data are being organized for presentation. It is believed that this study is the only one of its kind.

## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

Introductory Remarks

It was reported in the previous progress report that all research that was originally proposed was completed. A new task, the effect of damage on compressive buckling behavior, would be undertaken. Here we report the preliminary experiments on this task.

Damage Tolerance Tests

The highly redundant nature of the isogrid stiffening concept could render this type of construction tolerant to damage. An exploratory experimental study has been initiated to explore this question. The grid provides most of the bending stiffness for isogrid structure. It is sensible to consider damage to grid ribs, therefore, for stiffness critical, buckle resistant applications.

Two beam specimens with single backbone ribs were prepared and tested in compression. The tests were non-destructive with buckling loads determined by a stiffness plotting technique. The backbone ribs were progressively severed at nodal sites and retested. Degradation of buckling resistance was determined, therefore, in this manner.

The specimens lost approximately half of their buckling resistance with the first break in the backbone rib. With additional breaks, a smooth trend emerged tending toward buckling load values corresponding to the skin alone.

These preliminary tests confirmed the value of this type of investigation. Currently, panel tests are being planned. A semi-automated data acquisition has been assembled to facilitate the testing. The rapid degradation with the first rib break experienced in the beams will not occur here. The panels offer rib redundancy.

### Reporting of Program Results

Most of our effort is directed toward preparing a report of this phase's results. In a modified form, it will constitute the Ph.D. thesis of A. D. Reddy.

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Quarterly Progress Report No. 9  
1 July 1980 - 30 September  
AFOSR Contract F49620-78-C-0085

L. W. Rehfield and S. N. Atluri  
College of Engineering  
Georgia Institute of Technology  
Atlanta, Georgia 30332

## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the Direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.



Phase A:

"Computational Methods for Fracture and Fatigue Analyses"

and

"Complementary Energy Approaches for Nonlinear Stability"

S.N. Atluri

Center for the Advancement of Computational Mechanics  
School of Civil Engineering  
Georgia Institute of Technology

In the previous three months (7/1/80 to 10/1/80) the following research was accomplished.

I. Stress and Fracture Analysis of Angle-Ply Laminates:

The development and satisfactory evaluation of special laminated hole elements, based on a stress-hybrid finite element model and a complementary energy approach, has been completed. The set of elaborate results obtained are being documented for publication in archival literature.

The following is a cumulative list of publications and/or presentations under this item of research.

- (i.1) "Cracks in Laminates", Presentation 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (i.2) T. Nishioka and S.N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R.H. Sierakowski), University of Florida Press, pp 95-101.
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- (i.5) T. Nishioka and S.N. Atluri, "Analysis of a Through-Crack in  $(-45^{\circ}/+45^{\circ}/+45^{\circ}/-45^{\circ})$  Laminate Using a Complementary Energy Principle", paper submitted to Journal of Engineering Fracture Mechanics, August 1979.
- (i.6) S.N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates", Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.

- (i.7) T. Nishioka and S.N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates" 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference, Seattle, Wash., May 1980, pp 295-302.
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- (i.11) T. Nishioka and S.N. Atluri, "Studies on Fracture of Angle-Ply Laminates", accepted for presentation and publication, 4th International Conference on Fracture, Cannes, France, March 1981.
- (i.12) T. Nishioka and S.N. Atluri, "Assumed Stress Finite Element Analysis of Through-Cracks in Angle-Ply Laminates", AIAA Journal, to appear, September 1980, 45 pages.
- (i.13) T. Nishioka and S.N. Atluri, "Analysis of Through-holes in Composite Laminates: A Special Hole Element Approach" being submitted to AIAA Journal.
- (i.14) T. Nishioka and S.N. Atluri, "An Approximate 2-D Analysis of Stress Concentration Around Holes in Laminates", being submitted to Journal of Composite Materials.

## II. Fatigue and Fracture of Metallic Materials:

Analysis of through-thickness cracks in only one layer of an adhesively bonded metallic laminate, with or without debonding of the laminates around the crack, has been completed. The computer program based on this analysis has been satisfactorily verified. Parametric studies with variations of crack-length, debond-area, and material properties of the lamina, have been completed.

The above analysis and results are being documented in a paper being prepared for presentation and publication at the 22nd AIAA/ASME/ASCE/AHS Structures, Structural Dynamics Conference in Atlanta, Ga., April 81.

The following is a cumulative list of publications under this item of research.

- (ii.1) M. Nakagaki and S.N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II", AIAA Paper No. 1 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Missouri, April 1979, pp 221-232.

- (ii.2) M. Nakagaki and S.N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode - I Spectrum Loading", Proceedings Third International Conference on Mechanical Behavior of Materials, Cambridge University, United Kingdom, Pergammon Press, 12 pages.

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- (ii.3) S.N. Atluri, M. Nakagaki and K. Kathiresan, "Hybrid Finite Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics", in Engineering Application of the Finite Element Method, Vol. 1, Det Norske Veritas, Norway, 1979, pp 14.1-14.32.

Also to appear in Journal of Computers and Structures, 1980.

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- (ii.5) S.N. Atluri, T. Nishioka and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, by Moving Singular Elements", Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, AMD Vol. 25, pp 66-102.
- (ii.6) A.S. Kobayashi, J.S. Cheng, S.N. Atluri, A.F. Emery and W.J. Love, "Elastic-Plastic Analyses of a Three-Point Bend Specimen and a Fracturing Pipe", invited contributions, U.S.-Japan Seminar and Elastic-Plastic Fracture, Hyama, Japan, November 1979.
- (ii.7) T. Nishioka, R.B. Stonesifer, and S.N. Atluri, "Translating Singularity Method of Analysis of Rapid Crack Growth", invited paper, 2nd International Conference on Numerical Methods in Fracture, Swansea, U.K. July 1980.
- (ii.8) S.N. Atluri, Higher-order, Special and Singular Elements, Invited Paper, Chapter 4 in Finite Element Methods, published by ASME, NY, (to appear) 1980.
- (ii.9) T. Nishioka and S.N. Atluri, "Analysis of Cracks in Adhesively Bonded Metallic Laminates", prepared for presentation and publication at 22nd AIAA/ASME/ASCE/AHS SDM Conference, Atlanta, April 1981.
- (ii.10) M. Nakagaki and S.N. Atluri, "Elastic-Plastic Analysis of Fatigue Crack-Closure in Modes I and II", AIAA Journal, Vol. 18, No. 9, pp 1110-1118, Sept. 1980.

### III. Complementary Energy Approaches for Nonlinear Stability:

New results have been obtained for large-deformation and stability of plates using complementary energy method with symmetrized Biot stress and rigid rotations as variables. These results are being documented for publication.

The following is a cumulative list of publications under this item.

- (iii.1) S.N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis", Invited Contribution, International Conference on Theory Practice of Finite Elements, Chalmers Institute of Technology, Goteborg, Sweden, August 27-29, 1979.
- (iii.2) S.N. Atluri, "Finite Strain Inelasticity, Complementary Energy, and Finite Elements: Some Recent Computational Studies", Invited Paper, U.S.-Europe Conference on Finite Elements in Nonlinear Structural Mechanics, Bochum, West Germany, July 1980.
- (iii.3) S.N. Atluri, H. Murakawa and K.W. Reed, "Stability Analysis Via a New Complementary Energy Principle" presented at 2nd International Conference on Nonlinear Mechanics and Finite Elements, JIAFS and GWU, Washington, DC, October 1980. (Also to appear in Journal of Computers & Structures, Pergamon).
- (iii.4) H. Murakawa and S.N. Atluri, "Complementary Energy Analysis of Large Deformations and Post-Buckling of Plates" being prepared for presentation and publication at 22nd AIAA/ASME/ASCE/AHS SDM Conference, Atlanta, April 1981.
- (iii.5) S.N. Atluri, H. Murakawa, "Studies in Hybrid Methods: Fluid Flow and Elastic Stability" Invited Paper, 17th SES Meeting, Atlanta, Ga., Dec. 1980.
- (iii.6) S.N. Atluri and H. Murakawa, "Advances in Hybrid Methods in Non-linear Mechanics", International Symposium on Hybrid and Mixed Methods, Atlanta, Ga., April 1981.

## PHASE R1

### "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

#### Introductory Remarks

During this quarter, the work described below was performed. A primary task associated with the research at this time is the compilation, organization and preparation of theses, reports and papers which summarize the accomplishments made under this contract.

#### Hygrothermal Effects

Damping-frequency experiments on composite specimens in hygrothermal environments have been completed. Four types of specimens have been tested in five environmental states over a range of frequencies from 10-1000 Hertz. This completes a 3.5 year effort that produced a unique data base for graphite/epoxy composites. An important finding from the most recent work is that frequency effects are minor.

A summary of this work has been presented at the DOD/NASA Mechanics of Composites Review, Dayton, Ohio, October 28-30, 1980.

Two papers based upon this work will result. The references are

Rehfield, L. W., Briley, R. P., and Putter, S., "Dynamic Tests of Graphite/Epoxy Composites in Hygrothermal Environments," to be presented at the ASTM Symposium on Composites for Extreme Environments, Bal Harbor, Florida, November 11-12, 1980 and published in the proceedings.

Putter, S., Buchanan, D. L., and Rehfield, L. W., "Influence of Frequency and Environmental Conditions on Dynamic Behavior of Graphite/Epoxy Composites," submitted for presentation at the Sixth ASTM Conference on Composite Materials: Testing and Design, Phoenix, Arizona, May 12-13, 1981 and for publication in the proceedings.

### New Bending Theory

Most of the work in this area has been directed toward preparing results for publication and presentation. The following two papers are being prepared and abstracts have been written:

Rehfield, L. W., and Murthy, P. L. N., "Structural Dynamics of Beams: A New Theory and Applications," submitted for presentation at the 1981 AIAA Dynamics Specialists Conference, Atlanta, Georgia, April 1981 and for publication in the proceedings.

Rehfield, L. W., and Valisetty, R. R., "Bending of Plates: A Refined Theory and Comparative Study," submitted for presentation at the 22nd AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Atlanta, Georgia, April 1981 and for publication in the proceedings.

### Dynamic Behavior of Woven Composites

A comparative study of composites made from woven cloth prepreg and unidirectional tape prepreg has been completed. Dynamic experiments on beams were performed and stiffness and damping were determined. The behavior was found to be similar.

The results will be reported in

Putter, S., and Rehfield, L. W., "Dynamic Properties of Graphite/Epoxy Woven Cloth Composites in Bending," submitted for presentation at the 23rd Israel Annual Conference on Aviation and Astronautics and for publication in the proceedings.



## PHASE R2

### "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

#### Introductory Remarks

It was reported previously that all research that was originally proposed was completed. A new task, the effect of damage on compressive buckling behavior, has been undertaken. Here we report preliminary experiments on panels.

#### Damage Tolerance Tests

The highly redundant nature of the isogrid stiffening concept renders this type of construction tolerant to damage. An experimental study on beams has been completed which explored this question. The grid provides most of the bending stiffness for isogrid structure. It is sensible to consider damage to grid ribs, therefore, for stiffness critical, buckle resistant applications.

Preliminary tests are being conducted on the three remaining small panels to determine the influence of rib damage on compressive buckling resistance. On the basis of incomplete results on the first panel, it has been learned that damage to a single rib results in only a 4-12 percent reduction in buckling load. This confirms our expectations.

In anticipation of the final findings, an abstract has been prepared and an invited paper will be presented in Japan in January 1981. The complete reference is

Rehfield, L. W., and Reddy, A. D., "Damage Tolerance Studies on Continuous Filament Graphite/Epoxy Isogrid Structures", Joint US-Japan Conference on Composite Materials, Tokyo, Jan. 12-14, 1981.

A work-in-progress presentation with the same title will be presented at the AIAA/ASME/ASCE/AHS 22nd SDM Conference in Atlanta in April 1981.



### Reporting of Program Results

Most of our efforts is directed toward preparing a report of this phase's results. In a modified form, it will constitute the Ph.D. thesis of A. D. Reddy. The following paper is also in preparation

Rehfield, L. W., and Reddy, A. D., "Design Information for Continuous Filament Advanced Composite Isogrid Structure," to be presented at the Fifth DOD/NASA Conference on Fibrous Composites in Structural Design, New Orleans, January 27-29, 1981.

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

Annual Scientific Report  
1 July 1978 - 30 June 1979  
AFOSR Contract F49620-78-C-0085

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## INTRODUCTION

This research program is divided into three phases of activity, which are denoted A, R1 and R2. Phase A, "Computational Methods for Fatigue and Fracture Analysis", is directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

Phase A  
Computational Methods for Fracture and Fatigue Analyses  
and  
Complementary Energy Approaches for Nonlinear Stability

S. N. Atluri

In the 12 month period, 7/1/78 - 6/30/79, the following research was accomplished.

1. Stress and Fracture Analysis of Angle-Ply Laminates:

A new assumed stress hybrid finite element method, based on a complementary energy principle, has been developed for stress as well as fracture analyses of angle-ply laminates. The loading cases can include inplane as well as general bending loads. In this method, the fully three-dimensional stress-state (including the transverse shear and normal stresses) in each lamina is accounted for; the mixed-mode stress and strain singularities near the crack front, the intensities of which vary within each ply in the thickness direction of the laminate, are embedded in Special elements near the crack-front; the inter-laminar traction reciprocity conditions are satisfied a priori; and the interelement traction reciprocity conditions in the finite element mesh are satisfied through a Lagrange Multiplier technique. In the present procedure, each finite element consists of the entire stack of lamina, each of which is treated as an anisotropic medium. The mixed-mode stress-intensity factors  $K_I(t)$ ,  $K_{II}(t)$ , and  $K_{III}(t)$  in each lamina (and their variation with thickness  $t$ ) are computed directly in the present procedure.

In the above development, a new theoretical method to study the effects of the free-surfaces on stress-intensity factors has been formulated. In the analytical solutions for embedded flaws, in either isotropic or anisotropic media, it is found that the asymptotic stress solution obeys the plane-strain constraint. However, when a crack intersects a stress-free surface, say at a right angle, it is noted that the solution in the vicinity of the free surface must necessarily be of a plane-stress type. The transition from this plane-stress state to a plane-strain state in the interior region of the crack, has been a subject of much controversy in literature. In the present research, the constraints of either plane-stress or plane-strain along the crack-border have been removed a priori. Instead, an asymptotic (singular) stress solution, for cracks in general multi-layer anisotropic media, that satisfies the equilibrium equations only, a priori, has been derived. The principle of complementary energy is then used to force the above stress-solution to satisfy compatibility of deformation. The numerical solution thus automatically predicts conditions of plane-stress or plane-strain along the crack border, as the case may be.

Excellent results for several problems of cracks in multilayered anisotropic or isotropic laminates have been obtained.

Modifications to the above procedures, to enable an accurate analysis of cracks near fastener holes in adhesively bonded metallic laminates have been considered.

## 2. Two Dimensional Analysis of Heterogeneous Systems:

Several two dimensional problems of cracks in lamina wherein the matrix and fibers are modeled individually have been solved. The general case of a crack running at an angle to the fiber orientation was treated. Explorative studies into the feasibility of analysis of stable growth of a crack oriented at an angle to the fiber-direction and growing across the fiber, have been made.

## 3. Continuation of Ongoing Research into Fatigue and Fracture of Metallic Materials:

Analysis of fatigue crack growth of cracks in panels subject to Mode I and Mode II type cyclic loading have been performed. The cyclic load spectra that were considered are: (i) constant amplitude zero to tension, (ii) high-to-low spectrum, (iii) low-to-high spectrum and (iv) single overload in an otherwise constant amplitude spectrum. Quantitative results were obtained for effects of fatigue crack closure on the acceleration or retardation of crack growth due to stress-interaction. Some highlights of the conclusions from these studies were: (a) depending on the overload stress ratio, fatigue crack-growth can actually be stopped through the application of a single overload; (b) that fatigue cracks do not close under pure Mode II loading, unlike in the Mode I case. Thus, when considering fatigue growth of cracks in mixed-mode loading and in considering the effects of crack-closure on the growth rate of cracks under this loading, only Mode I component needs to be analyzed.

Analyses of the problem of fatigue growth of cracks near fastener holes have also been performed. Results have been obtained for sizes of plastic zones near fastener holes subjected to various levels of cold-working. The significant results from the analyses of fatigue-crack-growth near the fastener holes are: (a) the process of cold-working results in retardation of fatigue growth of cracks in cases where the crack is completely embedded within the plastic zone created by cold-working and also when the crack is longer than the plastic zone due to cold-working; (b) in both the forementioned cases, the retardation effects are more significant during the initial phase of fatigue crack-growth than after sufficient number of load-cycles.

## 4. Complementary Energy Approaches for Nonlinear Stability:

Basic exploratory work has been performed to derive the beam, plate and shell equations starting with the a priori polar-decomposition of the deformation gradients into rigid-rotations and stretch. These equations which are being developed in general curvilinear coordinates represent a novel departure in the engineering theory of plates and shells. Since a significant number of nonlinear problems involve large rotations but small strains, the treatment of rigid-rotation directly as an independent variable in the kinematics, as well as in the complementary energy principle, would lead to significant improvements in the analysis of these problems. Analytical results generated so far, for a variety of beam stability problems, appears to confirm this view.

## Papers Published and Presented

- (1) "Cracks in Laminates" presentation, 4th USAF/Army/Navy/NASA Composites Review Meeting, Dayton, Ohio, October 31, 1978.
- (2) T. Nishioka, and S. N. Atluri, "An Efficient Assumed Stress Finite Element Procedure for the Analysis of Multilayer Anisotropic Laminates" in Recent Advances in Engineering Science, (Ed. R. H. Sierakowski), University of Florida Press, pp. 95-101.
- (3) T. Nishioka and S. N. Atluri, "Fracture-Stress Analysis of Through-Cracks in Angle-Ply Laminates: An Efficient Assumed Stress Finite-Element Approach - I" AIAA Paper No. 79-0801, Proc. 20th AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, St. Louis, Mo., April 1979, pp. 315-326.  
The above is also under review for publication in AIAA Journal.
- (4) S. N. Atluri, M. Nakagaki, and K. Kathiresan, "Numerical Studies of Some Problems of Engineering Fracture Mechanics: Fatigue Crack Growth and Surface Flaws," International Conference on Fracture Mechanics in Engineering Application, Bangalore, India, March 26-30, 1979, p. 59.
- (5) T. Nishioka and S. N. Atluri, "Analysis of a Through-Crack in  $(-45^\circ/+45^\circ/+45^\circ/-45^\circ)$  Laminate Using a Complementary Energy Principle," paper being submitted to Journal of Engineering Fracture Mechanics, August, 1979.
- (6) S. N. Atluri, "Analytical Modeling of Free-Surfaces in 3-D Cracks in Laminates," Invited Presentation, Society for Experimental Stress Analysis, Spring Meeting, (Special Research Session), May 25-29, 1979.
- (7) T. Nishioka, and S. N. Atluri, "An Assumed-Stress Finite Element Approach to Analysis of Edge-Effects and Holes in Laminates," being prepared for presentation and publication, 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Meeting, Seattle, Washington, 1980.
- (8) T. Nishioka and S. N. Atluri, "A New Approach to Estimate Free-Surface Effects for Surface Flaws," being prepared for submission to International Journal of Numerical Methods in Engineering.
- (9) M. Nakagaki, and S. N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II," AIAA Paper No. 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Mo., April 1979, pp. 221-232.  
This is also under review for publication in AIAA Journal.
- (10) M. Nakagaki, and S. N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode-I Spectrum Loading," Proc. Third International Conference on Mechanical Behaviour of Materials, Cambridge University, United Kingdom, Pergamon Press, 12 p.  
The above is also scheduled to appear in the Journal of Fatigue, vol. 1, 1979.
- (11) S. N. Atluri, M. Nakagaki, and K. Kathiresan, "Hybrid Finite-Element Analysis of Some Nonlinear and 3-D Problems of Engineering Fracture Mechanics," in Engineering Application of the Finite Element Method, vol. 1, Det Norske Veritas, Norway, 1979, pp. 14.1 - 14.32.
- (12) S. N. Atluri, "Numerical Modeling of Crack Growth," Invited Contribution, to appear in Proc. 3rd ASCE, Engineering Mechanics Specialty Conference, Austin, Texas, September 16-19, 1979.

- (13) S. N. Atluri, T. Nishioka, and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, By Moving Singular Elements," Invited Contribution, to appear in Nonlinear and Dynamic Fracture, ASME Special Publication, to appear December, 1979, 32 pages.
- (14) S. N. Atluri, "Complementary Rate Principles for Nonlinear Structural Analysis," Invited Contribution, International Conference on Theory and Practice of Finite Elements, Chalmers Institute of Technology, Gotenborg, Sweden, August 27-29, 1979.



## PHASE R1

## "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

Introductory Remarks

As the research work unfolded during the year, two dominant areas emerged as the primary foci. The first, hygrothermal effects on resin matrix composite structures, is a continuation of work begun under a previous grant. The second, development of a new bending theory, is a serendipitous outgrowth of work related to the first. While the first task has been dominantly experimental in nature, the latter is exclusively theoretical up to this point.

Hygrothermal Effects

This work has been directed toward determining the influence of moisture absorption and elevated temperature environments on the dynamic behavior of resin matrix composites in bending. An extensive series of beam vibration experiments have been performed. Specimens of four distinct ply layups have been tested at four different temperatures. These experiments have produced nearly 2000 distinct data values over a two year period.

An additional experimental study has been performed which has turned out to be of great interest. By virtue of some difficulties with instrumentation some experiments were conducted on aluminum beams. A closer examination of the resulting data prompted a comparative study of hygrothermal effects on graphite/epoxy with the effect of temperature alone on two aluminum alloys. The data indicate that damping changes associated with hygrothermal effects on the composites are of the same order of magnitude as those produced by elevated temperature for the aluminum alloys tested. The only new consideration for graphite/epoxy composites

that must concern the designer is the increased stiffness reduction in matrix controlled modes of response.

This research appears to be the only work of its kind. Stiffness and damping data were supplied to persons at Rockwell International for their use in a space shuttle vibration problem. Currently an effort is being made to further analyze the vibration data, which will all appear in R. P. Briley's doctoral thesis. In particular, it is important to estimate the contributions to measured damping due to aerodynamic effects, moisture condensation and testing technique. This activity has been undertaken in response to suggestions received from AFML researchers who have a strong interest in this work.

Portions of the above work were presented at two meetings during the year. The appropriate references are

Rehfield, L. W., and Briley, R. P., "Dynamic Environmental Effects on Aerospace Materials", ASCE Preprint 3328, presented at the ASCE Annual Convention, Chicago, Ill., October 16-20, 1978.

Rehfield, L. W., and Briley, R. P., "A Comparison of Environmental Effects on Dynamic Behavior of Graphite/Epoxy Composites With Aluminum Alloys", ASME Paper 78-WA/Aero-10, presented at the ASME Winter Annual Meeting, San Francisco, December 10-15, 1978.

In addition, an invited presentation was given at the USAF Workshop on Damping in Composite Structural Materials held at MIT on June 15, 1979. (The above suggestions for further data analysis were received at this meeting.) The complete citation for this presentation is

Rehfield, L. W., "A Synopsis of Research on Vibration Behavior of Advanced Composites in Hygrothermal Environments", USAF Workshop on Damping in Composites Structural Materials, MIT, Cambridge, Massachusetts, June 15, 1979.

As a result of our experience and the timely importance of the sub-

ject, a survey of the status of hygrothermal testing of resin matrix composites as it has developed over the last four years was initiated. Originally the intent was to coordinate our study with AFFDL research at Grumman Aerospace and the AMFL sponsored programs at General Dynamics - Fort Worth and Lockheed California Company and to attempt to identify the future directions in this type of testing, as well as problems to be faced. The scope was subsequently expanded. With encouragement from Dr. C. C. Chamis of NASA Lewis Research Center, a survey paper will be prepared for presentation at the upcoming ASTM Conference on Composite Materials Testing and Design Allowables (October 1979, Dearborn, Michigan). It is planned to include USN and NASA sponsored activities in the survey also.

#### New Bending Theory

In addition to the experimental work, analytical studies of the influence of hygrothermal effects on the design of stiffness critical composite structures were initiated. Significant new theoretical developments emerged from this work. The original intent was to study the influence of hygrothermal effects on the design of stiffness critical composite structures; vibration and buckling behavior were to be emphasized. Mr. P. L. N. Murthy, a graduate research assistant working on this task, discovered some background theoretical problems that required treatment before the bulk of the study could proceed. This was a stroke of good fortune! It has lead to a significant breakthrough in engineering bending theory.

Hygrothermal effects in resin matrix/graphite composites manifest themselves by amplifying the importance of matrix controlled behavior modes. Transverse shear deformations, for example, play a greater role in the response to environmental loading. This effect was the first to be considered.

Shear deformation theory for homogeneous, isotropic beams originated in a paper by S. P. Timoshenko published in 1921. Since then, there have been some refinements and extensions to plates and shells, but no conceptual differences. Our recent reassessment of this theory has shown that there are two additional effects that (a) are the same order as transverse shear and (b) have never been included in any engineering bending theory. These effects are called non-classical bending and transverse normal strain effects.

The above observations have lead to the development of an engineering bending theory that accounts for the two effects mentioned above. A complete static theory for the bending of homogeneous, isotropic beams was developed first. Predictions using this theory agree exactly with elasticity solutions for several distributed loading cases. The theory has been extended in two directions. A first approximation dynamic theory has been formulated. Also, a corresponding theory for orthotropic beams, a logical step in the direction of a lamination theory for composites, has been defined.

An evaluation of the first approximation dynamic theory for homogeneous, isotropic beams has been completed. Extremely fine agreement with the elasticity theory solution for a rectangular slab has been obtained. Comparison with the fully three-dimensional solution for a solid circular bar indicates some additional refinement for non-rectangular cross sections is needed. The nature of the discrepancy for circular cross sections is being investigated further.

Static and dynamic agreement of solutions obtained using the orthotropic theory with available elasticity solutions is quite good. The sensitivity of the dynamic theory to cross sectional shape has not arisen in this context due to the absense of elasticity solution benchmarks

for orthotropic materials. A resolution for the isotropic material case, however, can be carried over to the orthotropic case in all likelihood.

All in all, this work is progressing well and should constitute a substantial contribution to structural theory. Because of the nature of this work, public exposition has been purposely delayed until a high degree of confidence and maturity is achieved.

## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

Introduction

Continuous filament advanced composite isogrid is a stiffening concept for flight vehicle structure that was developed in a company-funded manufacturing technology program at McDonnell Douglas. Pioneering evaluation of this concept in load-bearing structure was conducted under a previous AFOSR contract. Flat panels were manufactured by McDonnell Douglas and tested under compression at Georgia Tech.

Work on the evaluation of isogrid structures continues under the present contract. Strength and stiffness information has been gathered from elements cut from the three original, large flat panels. Considerable progress toward understanding the behavior of isogrid structures has been made.

Overview of the Program

Under contract F49620-77-C-0077, three large panels 10.5 inches by 21.0 inches of isogrid construction were designed and manufactured from graphite/epoxy. The manufacturing approach utilized weaving and braiding techniques in conjunction with an elastomeric tooling concept. A non-destructive stiffness plotting technique was used to experimentally study the compressive buckling behavior of these panels with clamped loaded edges and free unloaded edges. An extrinsic stiffening concept was employed to study the second (next higher) buckling events.

Following buckling studies of the three original panels, each panel was cut into four smaller panels. In addition, a center rib remaining from this sectioning process was used to create short compression element specimens. Buckling tests of the small panels in their fundamental



mode were then conducted.

The buckling data showed considerable scatter among both the large panels and the small ones cut from the same parent panels. Inconsistency in manufacturing was clearly identified as the primary reason for this scatter. Never-the-less, considerable understanding of buckling behavior emerged. This insight has been reported in the final AFOSR report submitted for contract F49620-77-C-0077. When looked at in the proper way, correlation of the experimental buckling data with existing theory is, on a relative basis, excellent. A major step had been taken, therefore, toward resolving the issues surrounding the potential use of continuous filament advanced composite isogrid in stiffness critical applications. This was not a small accomplishment in view of the scatter of the test data.

While the first contract's activity was focused upon overall stiffness controlled modes of behavior, the current work is directed toward strength controlled behavior also and detailed correlation with theory. Since the buckling tests were, with only minor exceptions, non-destructive, the small panels are available for creating specimens for element tests. An extensive element test program, including chemical assessment of fiber content, has been planned and is well along toward completion. Also, the development of a shear deformation theory has been undertaken for isogrid structures. More details concerning these tasks follow.

#### Isogrid Element Specimens and Tests

Every effort has been made to extract the maximum amount of useful information for the existing panels and the short compression specimens described earlier. From each small panel, two beam specimens can be obtained. The beams are cut such that a "backbone" rib is situated along the length, approximately at midwidth. Each beam element is tested



extensively in some or all of the following experiments:

1. Buckling tests (nondestructive)
2. Stiffness evaluation (three-point bending tests (TPB tests) )
3. Stiffness evaluation (four-point bending tests (FPB tests) )
4. Flexural strength tests (TPB and FPB tests - after stiffness determination)
5. Geometric characterization by length measurements

It has been found that the three-point and four-point bending failure modes involve the isogrid ribs only. Consequently, portions of the ribs can be removed after bending tests are completed leaving an intact gauge length of undamaged skin in a beam specimens. Consequently, these elements can be tensile tested ex situ to determine skin properties.

When panels are sectioned to create beam specimens, two backbone ribs are trimmed away and are residual to the process. These ribs are then turned into ex situ rib specimens by the aerospace engineering shop by removing the attached skin. These rib specimens, then, have been instrumented with small strain gages and tensile tested ex situ to determine rib properties.

The short rib/skin compression specimens created from the residual center ribs (obtained by sectioning the large panels into small ones) were mounted in encapsuled ends by McDonnell Douglas Astronautics Company. Fifteen specimens, five from each of the three parent isogrid panels, have been tested in simple compression. Extensional stiffness was monitored on six specimens fabricated with resistance strain gages installed. Ultimate compressive strength and an indication of the extent nonlinear material effects influence behavior were determined by testing

to failure.

In addition to the above mechanical tests, preliminary fiber content tests have been completed on some rib and skin samples. A complete analysis of these test results awaits more testing. The fiber content is needed in order to correlate mechanical properties determined from element tests with predictions from micromechanics theory.

### Correlation and Analysis

The element compression tests provided data for correlation with theory. Measured rib and skin stiffness properties have been obtained from ex situ element tests; these property values have been used to predict stiffnesses for the element compression specimens. The predictions compare quite favorably with averaged measured values obtained in the compression tests. This is in the face of experimental scatter due to variations in specimen quality. Consequently, extensional behavior appears to be reasonably well understood.

There are many unresolved issues surrounding bending behavior. The picture is clouded by the data scatter and by nonuniformity in the beam specimens. It appears that ultimate bending moment, for example, is a more consistent quantity than predicted ultimate stress values calculated on the basis of theory with measured geometric and property values. This is a frustrating situation.

All the three and four point beam bending tests strongly show a pronounced length effect that suggests transverse shear is important. This is consistent with the fact that a transverse shear correction helped to correlate the panel buckling data. Further correlation of bending test data depends upon getting this matter satisfactorily resolved.

### Shear Deformation Theory

As indicated above, there are strong indications that transverse

shear effects are important for isogrid construction. Due to the unidirectional nature of the woven ribs, transverse shear stiffness is rather low. A simple, crude method of estimating its' effects was used in the correlation study. Since it is not a major undertaking, a shear deformation theory is currently being developed for the isogrid type of construction. The primary task here is developing a rational means for determining the transverse shear stiffness. This is coordinated with the bending theory development described in Phase R1.

#### Closing Remarks

The efforts to understand and correlate data with theory are frustrated by two things. The specimens vary widely in quality due to the developmental nature of the manufacturing process for this type of structure. Also, the number of available specimens is small so that statistical methods, in view of the observed data scatter, are not reliable for such small sample sizes. Every bit of information and insight, however, will be gleaned from the specimens that can be. The structural concept is proven and should not be abandoned because the manufacturing process requires further development.

Isogrid research results have been presented in the following two papers:

Rehfield, L. W., Deo, R. B., and Renieri, G. D., "Continuous Filament Advanced Composite Isogrid: A Promising Structural Concept", presented at the Fourth Conference on Fibrous Composites in Structural Design", San Diego, November 14-17, 1978. (Proceedings to be published.)

Rehfield, L. W., Deo, R. B., and Lehman, J. K., "Buckling of Continuous Filament Advanced Composite Isogrid Wide Columns in Axial Compression," AIAA Paper No. 79-0804, presented at the 20th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, St. Louis, April 4-6, 1979.

FAILURE PROCESSES IN  
ADVANCED COMPOSITE STRUCTURES

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## INTRODUCTION

This research is divided into four phases of activity, which are denoted A1, A2, R1 and R2. Phases A1, "Computational Methods for Fatigue and Fracture Analysis" and A2, "Complementary Energy Approaches for Non-linear Stability," are directed by Prof. S. N. Atluri, School of Civil Engineering. Phases R1 and R2 are under the direction of Prof. L. W. Rehfield, School of Aerospace Engineering. Phase R1 is entitled "Static and Dynamic Behavior of Advanced Composite Structures". Phase R2, "Behavior of Advanced Composite Isogrid Structures", is an effort involving cooperation between the School of Aerospace Engineering and McDonnell Douglas Astronautics Company-St. Louis.

For convenience, this report contains a separate section devoted to each phase.

## PHASE A1

## "Computational Methods for Fracture and Fatigue Analyses"

S. N. Atluri

Objectives:

The objectives of this phase of research were: (1) to develop efficient and accurate finite element procedures for stress and fracture analyses of angle-ply laminates, (2) to develop efficient procedures for analysis of fatigue crack growth and fracture in homogeneous metallic materials, and (3) to develop analysis procedures for cracks near fastener holes in adhesively bonded metallic laminates.

Accomplishments:

A new assumed stress hybrid finite element method, based on a complementary energy principle, has been developed for stress as well as fracture analyses of angle-ply laminates. The loading cases can include inplane as well as general bending loads. In this method, the fully three-dimensional stress-state (including the transverse shear and normal stresses) in each lamina is accounted for; the mixed-mode stress and strain singularities near the crack front, the intensities of which vary within each ply in the thickness direction of the laminate, are embedded in special elements near the crack-front; the inter-laminar traction reciprocity conditions are satisfied a priori; and the interelement traction reciprocity conditions in the finite element mesh are satisfied through a Lagrange Multiplier technique. In the present procedure, each finite element consists of the entire stack of lamina, each of which is treated as an anisotropic medium. The mixed-mode stress-intensity factors  $K_I(t)$ ,  $K_{II}(t)$ , and  $K_{III}(t)$  in each lamina (and their variation with thickness  $t$ ) are

computed directly in the present procedure.

In the above development, a new theoretical method to study the effects of the free-surfaces on stress-intensity factors has been formulated. In the analytical solutions for embedded flaws, in either isotropic or anisotropic media, it is found that the asymptotic stress solution obeys the plane-strain constraint. However, when a crack intersects a stress-free surface, say at a right angle, it is noted that the solution in the vicinity of the free surface must necessarily be of a plane-stress type. The transition from this plane-stress state to a plane-strain state in the interior region of the crack, has been a subject of much controversy in literature. In the present research, the constraints of either plane-stress or plane-strain along the crack-border for cracks in general multi-layer anisotropic media, that satisfies the equilibrium equations only, a priori, has been derived. The principle of complementary energy is then used to force the above stress-solution to satisfy compatibility of deformation. The numerical solution thus automatically predicts conditions of plane-stress or plane-strain along the crack border, as the case may be.

Analysis procedures were also developed for problems of stress-concentration near fastener holes in angle-ply laminates. In this approach, analytical asymptotic solutions for an elliptical hole in a general two-dimensional anisotropic solid, subjected to far-field tensile stresses along the principal directions of material orthotropy as well as far-field shear stresses, are employed. From these two-dimensional solutions, a three-dimensional asymptotic solution which accounts for the variation of the stress-concentration factors through the thickness of the lamina (and hence of the laminate), and which accounts for the transverse shear and normal stresses, is generated by integration through the thickness of the laminate. The computer coding based on this approach has been



successfully completed. The obtained results for four ply and six ply symmetric angle-ply laminates agree excellently with those in prior literature and more over indicate the relative efficiency and accuracy of the presently developed procedure.

Thus the above approaches lead to multilayer, 3-dimensional special "crack" and "hole" elements for analysis of cracks and holes in angle-ply laminates. The unique features of these elements are that the crack-surface and hole-surface conditions are satisfied exactly a priori, and that stress-intensity factors, the stress-concentration factors, and their variation along the thickness direction of each lamina are solved for directly.

Based on the insights gained from the above 3-dimensional analyses, 2-dimensional procedures to estimate the stress-intensity and stress-concentration factors approximately, were also developed. These 2-dimensional estimation procedures were found to work remarkably well in a variety of carefully chosen test cases, as far as the inplane stresses and deformations are concerned. However, to predict delamination failures, in which case an accurate knowledge of interlaminar normal and shear stresses are necessary, the presently developed 3-dimensional analysis procedures are mandatory.

Several two dimensional problems of cracks in lamina wherein the matrix and fibers are modeled individually have been solved. The general case of a crack running at an angle to the fiber orientation was treated. Explorative studies into the feasibility of analysis of stable crack growth of a crack oriented at an angle to the fiber-direction and growing across the fiber, have been made.

Using the multilayer 'hole' and 'crack' elements as described above, analyses of cracks near fastener holes in adhesively bonded metallic

laminates have been performed. Several parametric studies were conducted, with the parameters that were varied include: (i) relative thickness of adherends and adhesive, (ii) relative material properties of adherend and adhesive, (iii) ratio of hole diameter to laminate thickness, (iv) ratio of crack length to hole diameter, (v) cracks in one of the metallic laminas, with and without debonding of the lamina near the crack.

Under objective (3) of Phase A1, the following studies were accomplished.

Analysis of fatigue crack growth of cracks in panels subject to Mode I and Mode II type cyclic loading have been performed. The cyclic load spectra that were considered are: (i) constant amplitude zero to tension, (ii) high-to-low spectrum, (iii) low-to-high spectrum and (iv) single overload in an otherwise constant amplitude spectrum. Quantitative results were obtained for effects of fatigue crack closure on the acceleration or retardation of crack growth due to stress-interaction. Some highlights of the conclusions from these studies were: (a) depending on the overload stress ratio, fatigue crack-growth can actually be stopped through the application of a single overload: (b) that fatigue cracks do not close under pure Mode II loading, unlike in the Mode I case. Thus, when considering fatigue growth of cracks in mixed-mode loading and in considering the effects of crack-closure on the growth rate of cracks under this loading, only Mode I component needs to be analyzed.

Analyses of the problem of fatigue growth of cracks near fastener holes have also been performed. Results have been obtained for sizes of plastic zones near fastener holes subjected to various levels of cold-working. The significant results from the analyses of fatigue-crack-growth near the fastener holes are: (a) the process of cold-working results in retardation of fatigue growth of cracks in cases where the crack is completely

embedded within the plastic zone created by cold-working and also when the crack is longer than the plastic zone due to cold-working; (b) in both the formentioned cases, the retardation effects are more significant during the initial phase of fatigue crack-growth than after sufficient number of load-cycles.

The methods of moving singular elements developed for the above analyses have been found to be extremely novel and useful in the analysis of more rapid crack propagation in which inertia plays a significant role. Work is underway in analyzing crack propagation under dynamic loading.

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- (5) T. Nishioka and S.N. Atluri, "Analysis of a Through-Crack in  $(-45^{\circ}/+45^{\circ}/+45^{\circ}/-45^{\circ})$  Laminate Using a Complementary Energy Principle", paper submitted to Journal of Engineering Fracture Mechanics, August 1979.
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- (12) T. Nishioka and S.N. Atluri, "Assumed Stress Finite Element Analysis of Through-Cracks in Angle-Ply Laminates", AIAA Journal, to appear, September 1980, 45 pages.
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- (15) M. Nakagaki and S.N. Atluri, "An Elastic-Plastic Analysis of Fatigue Crack Closure in Modes I and II", AIAA Paper No. 1 79-0758, 20th AIAA/ASME/ASCE/AHS DSM Conference, St. Louis, Missouri, April 1979, pp 221-232.
- (16) M. Nakagaki and S.N. Atluri, "Fatigue Crack-Closure and Delay Effects Under Mode - I Spectrum Loading", Proceedings Third International Conference on Mechanical Behavior of Materials, Cambridge University, United Kingdom, Pergammon Press, 12 pages.

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- (23) T. Nishioka and S.N. Atluri, "Analysis of Cracks in Adhesively Bonded Metallic Laminates", prepared for presentation and publication at 22nd AIAA/ASME/ASCE/AHS SDM Conference, Atlanta, April 1981.
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## PHASE A2

## "Complementary Energy Approaches for Nonlinear Stability"

S. N. Atluri

Objectives:

The objectives of this phase of research were: (1) to develop appropriate complementary energy principles, involving the unsymmetric first Piola-Kirchhoff stress tensor and rotation tensor, for finite deformation of and stability analyses of beams, plates, and shells, (2) to develop the appropriate complementary energy density expressions for beams, plates, and shells in terms of first Piola-Kirchhoff stress resultants, and moment resultants, invoking plausible deformation hypotheses, and (3) to solve certain chosen cases of large-deformation and post-buckling of structural members.

Accomplishments:

Making a fundamental departure from the currently well-established beam, plate, and shell theories, new approaches have been developed for developing the beam, plate, and shell equations based on the a priori polar-decomposition of deformation into pure stretch and rigid rotation of material elements. The plausible approximations to the deformation gradient for these "thin" bodies are thus reduced to appropriate approximations for the stretch tensor and the rigid rotation tensor. Thus for instance, the originally 3-dimensional stretch tensor is reduced to a 2-dimensional one for the case of plates and shells, and the variation of this 2-dimensional stretch tensor is assumed appropriately as being linear or quadratic in the thickness direction. Moreover, the rotation tensor which, in general, may have a 3-dimensional variation, now becomes only a function of only mid-surface coordinates for plates

and shells. Likewise, the usual stress-resultants and stress-couples of the well-known plate and shell theories are now reduced and redefined in terms of the first Piola-Kirchhoff stress-resultants and stress-couples, or the Jaumann-type stress resultants and stress-couples.

Using the stretch tensor, rotation tensor, displacement vector, the first Piola-Kirchhoff stress-resultants and stress-couples as variables, a general variational principle of the Hu-Washizu type is developed. This general variational principle leads to a new set of plate and shell equations in terms of the forementioned variables. These new equations offer some fundamentally novel advantages for finite-deformation and buckling analysis of structures, over the currently popular ones.

Expressions for complementary energy density of plates and shells, in terms of first-Piola-Kirchhoff stress-resultants and stress-couples are newly developed. Using this, and satisfying the force equilibrium equations a priori, certain new and novel complementary energy principles for plates and shells are developed from the above mentioned general variational principle.

The above complementary energy principle was cast into an appropriate incremental form. By introducing the interelement stress-resultant reciprocity as an a posteriori constraint, a modified variational principle, from which a hybrid-finite element procedure can be developed, was generated.

The above procedures have been successfully tested in the cases of finite deformation and post-buckling of plates. These results do point to remarkable advantages of the presently developed procedures over the existing ones. This exciting line of new inquiry into plate and shell behavior is ongoing.



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- (2) S.N. Atluri, "Finite Strain Inelasticity, Complementary Energy, and Finite Elements: Some Recent Computational Studies", Invited Paper, U.S.-Europe Conference on Finite Elements in Nonlinear Structural Mechanics, Bochum, West Germany, July 1980.
- (3) S.N. Atluri, H. Murakawa and K.W. Reed, "Stability Analysis Via a New Complementary Energy Principle", presented at 2nd International Conference on Nonlinear Mechanics and Finite Elements, JIAFS and GWU Washington, DC, October 1980. (Also to appear in Journal of Computers & Structures, Pergamon).
- (4) H. Murakawa and S.N. Atluri, "Complementary Energy Analysis of Large Deformations and Post-Buckling of Plates" being prepared for presentation and publication at 22nd AIAA/ASME/ASCE/AHS SDM Conference, Atlanta, April 1981.
- (5) S.N. Atluri and H. Murakawa, "Studies in Hybrid Methods: Fluid Flow and Elastic Stability" Invited Paper, 17th SES Meeting, Atlanta, Georgia, December 1980.
- (6) S.N. Atluri and H. Murakawa, "Advances in Hybrid Methods in Non-linear Mechanics", International Symposium on Hybrid and Mixed Methods, Atlanta, Georgia, April 1981.

## PHASE R1

## "Static and Dynamic Behavior of Advanced Composite Structures"

L. W. Rehfield

Introductory Remarks

As the research work unfolded, two dominant areas emerged as the primary foci. The first, hygrothermal effects on resin matrix composite structures, is a continuation of work begun under a previous grant. The second, development of a new bending theory, is a serendipitous outgrowth of work related to the first. While the first task was dominantly experimental in nature, the latter is exclusively theoretical.

Objectives

Hygrothermal Effects on Dynamic Behavior. This research has three objectives. The first is to establish a data base to facilitate confident use of graphite/epoxy composites in dynamic applications. The second is to determine the extent to which viscoelastic factors influence dynamic behavior. This is reflected in the frequency dependence of response to time dependent excitation. The third objective is to determine the effects of moisture absorption and elevated temperature on dynamic behavior over a wide range of exciting frequencies.

These objectives have been accomplished by performing flexural vibration tests on graphite/epoxy beams. Dynamic behavior in the dry, room temperature state  $25^{\circ}\text{C}$  ( $77^{\circ}\text{F}$ ) is contrasted with the following four elevated temperature states:

- a.  $82^{\circ}\text{C}$  ( $180^{\circ}\text{F}$ ), dry
- b.  $60^{\circ}\text{C}$  ( $140^{\circ}\text{F}$ ), moisture saturated
- c.  $82^{\circ}\text{C}$  ( $180^{\circ}\text{F}$ ), moisture saturated
- d.  $93^{\circ}\text{C}$  ( $200^{\circ}\text{F}$ ), moisture saturated

New Bending Theory. Significant new theoretical developments emerged in the course of work done on hygrothermal effects. The original intent of the work was to study the influence of hygrothermal effects on the design of stiffness critical composite structures; vibration and buckling behavior were to be emphasized. It was discovered that there were some background theoretical problems that required treatment before the bulk of the study could proceed. This was a stroke of good fortune! It has lead to a significant breakthrough in engineering bending theory.

Hygrothermal effects in resin matrix/graphite composites manifest themselves by amplifying the importance of matrix controlled behavior modes. Transverse shear deformations, for example, play a greater role in the response to environmental loading. This effect was the first to be considered. The objective, therefore, was to lay the foundation for a bending theory which accounts appropriately for all the physical processes germane to the prediction of hygrothermal effects in resin matrix composites. The ultimate objective is a composite lamination theory.

#### Accomplishments

Hygrothermal Effects on Dynamic Behavior. This work has been directed toward determining the influence of moisture absorption and elevated temperature environments on the dynamic behavior of resin matrix composites in bending. An extensive series of beam vibration experiments have been performed. Specimens of four distinct ply layups have been tested at four different temperatures. The early experiments have produced nearly 2000 distinct data values over a two year period.

The specimens were manufactured by McDonnell Douglas Astronautics Company - St. Louis from Narmco 5208/T300 unidirectional tape. Four

distinct layup configurations have been tested as beams:  $[0]$ ,  $[\pm 45]$ ,  $[0, +45, 90, -45, 0, +45]_s$ , and  $[90]$ . They are each 12 plies thick and symmetric. The layup configurations are denoted A, B, C and D, respectively.

Considerable attention must be given to test conditions and testing technique. Since environmental effects are to be determined, vibration testing in a vacuum chamber at room temperature --- the usual means of determining damping --- could be used only for the reference state. All other tests have been performed in an environmental chamber at the proper temperature. Moisture saturation is achieved by immersion in a constant-temperature water bath for an extended period. Moisture absorption is monitored by periodically weighing the specimens.

The beams have been tested in cantilever fashion. In the early tests, transient excitation was used in order to insure short testing time and thereby minimize drying out effects. A tip mounted accelerometer was used to sense response. Damping was determined by the logarithmic decrement method, while fundamental natural frequency was found approximately by counting the number of response peaks in a given time interval.

The contribution of aerodynamic damping to the damping measured in tests is of considerable practical importance. The usual approach is to test specimens in a vacuum chamber, thereby eliminating aerodynamics altogether. This approach was not used because near in situ hygrothermal environmental tests were required. For small amplitudes, aerodynamic damping is proportional to amplitude. Therefore, tests have been conducted at several amplitudes. Damping vs. amplitude plots are constructed. Extrapolation to zero amplitude yields intrinsic material damping and aerodynamic damping. This technique is quite effective

and good estimates of the aerodynamic damping contributions to the results have been obtained. Material and air damping values are the proper magnitude and agree well with values published in the literature.

The other extraneous contribution to damping in the early tests is due to parasite effects associated with the accelerometer cable. The effects may be partially attributable to aerodynamics and partially due to mechanical deformation of the cable. Regardless of the cause, a reliable estimate of this contribution is desirable. Supplementary tests to determine the cable contribution were conducted. The results suggest that the cable damping is largely independent of amplitude. Therefore, simple adjustments to the data could be made.

Later, vibration experiments were performed which utilize electromagnetic noncontacting transducers and exciters. The primary purpose of these tests is to determine the influence of exciting frequency as well as environmental conditioning. The properties determined were an effective flexural modulus and damping for frequencies from 10-1000 Hertz.

In these tests, both excitation and response are created using electromagnetic noncontacting transducers. The tests are resonance tests. The natural frequency is varied by means of three approaches: mass addition, excitation of higher modes, and variation of unsupported beam length. This permits data in the 10-1000 Hertz range to be obtained. Small amplitude excitation is used exclusively in the elevated temperature tests to reduce aerodynamic damping effects to a minimum. Reliable aerodynamic damping estimates suggest that this influence is ignorable for the test conditions adopted. Damping is determined by suspending excitation and observing the decay of the response.

A and C specimens respond in a fiber controlled mode of behavior. Both environment and frequency have little effect on their stiffness. B and D specimens, however, exhibit matrix controlled behavior. Naturally, the response of these specimens is more sensitive to both environment and frequency; apparent stiffness decreases with the severity of the conditioning. Frequency effects on damping are small in all cases. Again, A and C specimen behavior is not greatly influenced by environmental conditioning. B specimen damping is affected by moisture, but very little by temperature. The damping of saturated D specimens increases sharply from 82°C to 93°C.

Overall, the damping results indicate that frequency effects are quite small in all cases. They are a bit greater for matrix controlled modes of response exhibited by the B and D specimens at the higher frequencies. At the same temperature, damping increases with moisture saturation. For dry specimens, however, by contrast, damping decreases with temperature.

This work provides an experimental data base for graphite/epoxy composites that describes the influence of moisture absorption, elevated temperature and frequency effects on dynamic behavior. The early work was cited as a major contribution to structural dynamics in 1978.\* The data has been used by engineers at Rockwell International in connection with a vibration problem with a composite panel for the space shuttle.

Dynamic Behavior of Woven Composites. Composites manufactured from woven cloth prepreg are microscopically and macroscopically different from their counterparts manufactured from unidirectional tape. Woven

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\* Research on vibration of composite cited in "Highlights 1978-Structural Dynamics," Astronautics and Aeronautics, Vol. 16, No. 12, December 1978, pp. 85-86-, 97.



cloth composites can be manufactured at significantly lower cost for many applications. This is due to the greater fiber volume laid down per unit of time. It is of interest, therefore, to contrast the behavior of woven and unidirectional tape composites.

This task consisted of dynamic testing corresponding beam specimens of woven cloth and unidirectional tape and documenting the observed differences. Also, exploratory hygrothermal effect studies were planned, depending upon the outcome of dry, room temperature dynamic tests.

Room temperature tests were performed in a vacuum chamber on two types of woven graphite/epoxy beam specimens. Also, similar tests were performed on [0] and [45] beam specimens of the same material system but of unidirectional tape prepreg. The testing procedure used was identical to that used in the later hygrothermal tests conducted using non-contacting transducers. It was found that woven cloth and unidirectional tape composites behave in the same manner. The similarity in damping characteristics is a significant finding. It permitted testing to be terminated without having to consider hygrothermal environments.

New Bending Theory. The scope of this work is restricted to planar bending situations. In its' present form, the theory applies to beams with thin rectangular cross sections which respond to planar bending in plane stress or to infinitely wide plates which respond in plane strain (cylindrical bending). Both isotropic and orthotropic materials are considered. Beams of orthotropic material are the simplest type of structures where composite material behavior can be studied.

Shear deformation theory for homogeneous, isotropic beams originated in a paper by S. P. Timoshenko published in 1921. Since then, there have been some refinements and extensions to plates and shells, but no



conceptual differences. Our recent reassessment of this theory has shown that there are two additional effects that are the same order as transverse shear and that have never been consistently accounted for in any previous engineering bending theory. These effects are called nonclassical bending and transverse normal strain effects.

The above observations have lead to the development of an engineering bending theory that accounts for the two effects mentioned above. A complete static theory for the bending of homogeneous, isotropic beams was developed first. Predictions using this theory agree exactly with elasticity solutions for several distributed loading cases. The theory has been extended in two directions. A first approximation dynamic theory has been formulated. Also, a corresponding theory for orthotropic beams, a logical step in the direction of a lamination theory for composites, has been defined.

An evaluation of the first approximation dynamic theory for homogeneous, isotropic beams has been completed. Extremely fine agreement with the elasticity theory solution for a rectangular slab has been obtained.

Consistent progress has been made in developing and applying the new bending theory. Emphasis on the application to beams has been given to simple static bending under uniform distributed loading and flexural vibrations. Both isotropic and orthotropic beams have been considered for the following four limiting cases of boundary restraint:

- (1) fully clamped
- (2) clamped-propped
- (3) simply supported
- (4) cantilever

Comparisons with Bernoulli-Euler and Timoshenko theories have been made.

A number of observations have been made:

- (1) natural frequencies and mode shapes predicted by the new theory are close to Timoshenko theory predictions;
- (2) bending moment values for fully clamped and clamped-propped beams depart significantly from Timoshenko theory predictions;
- (3) differences in predictions between our new theory and Timoshenko theory are more pronounced for orthotropic beams;
- (4) the additional factors accounted for in the new theory appear to be more significant for static applications.

One of the significant hygrothermal effects on resin matrix composites is the reduction in resin controlled stiffness properties. A preliminary assessment of the effect of hygrothermally induced transverse shear stiffness reduction has been made for unidirectionally laid up beams (which are orthotropic). Properties corresponding to three environmental conditions were used. Our findings suggest very significant changes in behavior at the higher moisture content levels.

The extension of the ideas behind the new bending theory to plates was explored. A new bending theory for isotropic plates has been developed. Several limited applications have been considered which facilitate comparisons with Reissner's plate equations. Thus far, findings have been analogous to those for the beam theory. This work is quite interesting and encouraging and has both fundamental and applied consequences.

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2. Rehfield, L. W., "Vibration Behavior of Advanced Resin Matrix Composites in Hygrothermal Environments," DOD/NASA Mechanics of Composites Review, Dayton, Ohio, October 31 - November 2, 1978.
3. Rehfield, L. W., and Briley, R. P., "A Comparison of Environmental Effects on Dynamic Behavior of Graphite/Epoxy Composites With Aluminum Alloy," ASME Paper 78-WA/Aero-10, presented at the ASME Winter Annual Meeting, San Francisco, December 10-15, 1978. See H6.
4. Rehfield, L. W., "A Synopsis of Research on Vibration Behavior of Advanced Composites in Hygrothermal Environments," USAF Workshop on Damping in Composite Structural Materials, MIT, Cambridge, Massachusetts, June 15, 1979.
5. Rehfield, L. W., and Valisetty, R. R., "A Survey of Hygrothermal Mechanical Test Methods for Graphite/Epoxy Composites," ASTM Symposium on Test Methods and Design Allowables for Fibrous Composites, Dearborn, MI, October 2-3, 1979.
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7. Putter, S., Buchanan, D. L., and Rehfield, L. W., "Influence of Frequency and Environmental Conditions on Dynamic Behavior of Graphite/Epoxy Composites," DOD/NASA Mechanics of Composites Review, Dayton, Ohio, October 28-30, 1980.
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## PHASE R2

## "Behavior of Advanced Composite Isogrid Structures"

L. W. Rehfield

Introductory Remarks

The search for efficient, light-weight aerospace structural concepts is a continuing process. One promising concept is isogrid. It is a simple discrete stiffening gridwork that employs a repetitive equilateral triangular pattern of ribs. The name "isogrid" is derived from the fact that the triangular grid exhibits isotropic properties in a gross or overall sense. The stiffening concept was developed by Dr. Robert R. Meyer of McDonnell Douglas Astronautics Company - Huntington Beach in 1964 under a NASA Contract.

Continuous filament composite isogrid (CFCI) is a type of construction developed by McDonnell Douglas Astronautics Company-St. Louis. The ribs of the grid are constructed of continuous unidirectional fibers by using a weaving process. It combines synergistically the efficiency of a stiffened structure with the superior properties of a composite material system in a manner consistent with automated manufacturing technology. Pioneering evaluation of this concept in stiffness critical applications was accomplished earlier. Flat panels were manufactured by McDonnell Douglas and tested under compression at Georgia Institute of Technology.

Work on the evaluation of isogrid structures continued under the present contract. Strength and stiffness information has been gathered from elements cut from the three original, large flat panels. Considerable progress toward understanding the behavior of isogrid structures has been made.

## Objectives

For the full potential of CFCI to be realized, basic data must be acquired and evaluated in conjunction with current theoretical models and analysis methods. A data base has been established and an extensive correlation study has been performed.

Both strength and stiffness controlled behavior have been studied. Three large panels, designed and manufactured by McDonnell Douglas Astronautics Company - St. Louis, were tested previously as wide columns in uniaxial compression with fixed loaded ends. A variety of element tests, including assessment of fiber content, have been performed to determine a maximum of information from the three original panels.

The objectives of the investigation described herein are to

- 1) explore fundamental behavioral processes for this new type of structure;
- 2) create an experimental data base to inspire confidence in the use of this concept;
- 3) correlate experimental data with theoretical predictions; and
- 4) identify and define problem areas where the state-of-the art in design analysis and manufacture of CFCI must be advanced.

## Accomplishments

An extensive element test program, including assessment of fiber content, has been completed. Compression and bending strength and stiffness data are presented that have been determined from elements cut from the original panels. This data permitted a comprehensive study to be performed which provides good correlation of theoretical predictions with the experimentally determined buckling loads. All issues are resolved and the degree of correlation is most satisfying. In addition,

interlaminar-type short beam shear tests and ex-situ tests of skin and grid ribs have been performed.

A wide variety of element types were prepared and tested extensively in bending, buckling, compression, and tension. Assessment of fiber content using chemical procedures was also carried out. A data base of strength and stiffness information is established which permits design with a measure of confidence.

Depending mainly on the manner in which the load is applied, the mechanical tests are grouped into the following five categories:

- 1) Element compressive tests
- 2) Bending tests - small panels and beam specimens
  - a. Three-point
  - b. Four-point
- 3) Compressive buckling tests - small panel and beam specimens
- 4) Ex-situ tensile tests - rib skin specimens
- 5) Short beam shear tests

The experimental data from all the tests was analyzed for consistency. The objective is to understand the observed behavior and correlate it with theoretical predictions whenever possible. An approximate scaling law based upon elementary physical reasoning is developed for relating the stiffness controlled behavior of beams to panels. Finally, the behavior of elements and panels has been predicted utilizing appropriate existing theories and the predictions compared with experimental results.

The experimental data obtained through this extensive, systematic research are self-consistent and show good agreement with theoretical predictions. The pronounced effect of transverse shear flexibility is



evident in the bend tests and in the correlation of buckling data. A scaling law is suggested which is useful in assessing buckling behavior of compressed CFCI structures. Manufacturing inconsistencies are clearly identified as the primary reason for the scatter in test data. Improvements in manufacturing technology must receive the highest priority if this structural concept is to be used in applications.

The element test data and results of the buckling correlation study are sufficient to establish a basis for design and utilization of continuous filament advanced composite isogrid. The most significant finding is the necessity of accounting for transverse shear flexibility in bending and buckling.

#### Papers, Reports and Presentations

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